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$S(z_p, z_p)$: Post-Structural Readings of Gödel's Proof

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Acknowledgement

At one time I was labelled a mathematical prodigy. But due to insufficient luck, talent or motivation I wasn't as successful as my teachers had hoped. Whatever was missing, it was definitely not the nowise short of excellent education I received from Prof. Vitali Milman and Prof. Efim Gluskin, who are directly responsible for those mathematical achievements I did attain, and for the kind of mathematical insight that some colleagues still credit me with.

Despite my growing impatience with mathematics, the transition to the humanities was slow and undecided. At first I had no tolerance for any but the most reductive materialist and formalist forms of thinking. I would like to acknowledge professors Amal Jamal, Daniel Dor and Lyat Friedman for making other ways of thinking accessible to me in the earlier stages of my explorations. The part of my personal and intellectual friend Yehonatan Alshekh in this transition was no less crucial.

The idea for the dissertation on which this book is based arose from two incidents. The first was a talk given by Dr. Dalit Baum in a context that I don't completely recall (and neither does she). She presented an alternative idea for how to teach mathematics to humanity students. Rather than present a host of pretty mathematical riddles and drawings (starring Mandelbrot sets and heart shaped curves) or giving a technical introductory freshman course (which would be quite pointless if given in isolation), she suggested that humanity students be guided in reading actual mathematical texts, applying their proficiency in textual analysis to explore these texts' horizons. The second incident was a classroom discussion in Prof. Adi Ophir's seminar on Derrida. While discussing Derrida's theory of signs, a student asked whether this theory would also apply to mathematical signs. It was clear to me that it would, and in a more obvious way than it does for other uses of signs.

The first drafts of chapter 2, which is the core of this essay, were produced for presentation in Prof. Adi Ophir's methodological seminar and the late Prof. Ruth Manor's logic seminar. The first was a supportive framework, which gave me free range to explore ideas that I might not have

been able to present anywhere else. The second was a small and analytically minded group of critical young researchers, who forced me to deal with Gödel's text hands-on without cutting any corners. I was warned that Prof. Manor would not quite tolerate my post-modern nonsense. As it turned out, she was not only supportive, but actually protective and encouraging. I wish I had the chance to learn more from her, and share with her my research in its more developed form.

By the time I decided to write down the philosophy Ph.D. dissertation on which this book is based, my choice of supervisors was clear. I don't know where I would be, if either of these two incredibly busy people would have turned me down. Prof. Anat Biletzki was the analytic critical mind I needed to keep me 'in line'; but what distinguishes her among such philosophers is a rare ability to indulge positions that she completely rejects, and come up with constructive reactions. Prof. Adi Ophir's part was primarily to be patient enough to teach me everything I needed to know in order to safely enter the *pardes* of post-structural thinking, and to guide me through its thicket. He was even willing to take the leap of faith, which would land him in the deep end of the formal logic pool for the benefit of seeing me through this project. Without their guidance this book would have been impossible for me to write; without their critical reading and remarks this book would have turned out far worse. Since the expertise of neither of my supervisors completely exhausts the entire range of this essay, its many faults are due entirely to me.

Various readers were generous enough to go over parts of this text. I would like to acknowledge Professors Claude Rosental, Eric Brian and Sabetai Unguru for their helpful comments. Prof. Unguru's remarks were especially useful, as they came from an opposition stance, which at the same time was kind enough not to prejudge this entire work as complete hogwash.

I would like to end this list of acknowledgements where it should have perhaps begun. My mother taught me to think. For her, and subsequently for me, thinking is something that should always be done from the point of view of someone else. My father tried to teach me how to be a *mentsh.* If I hadn't been so busy thinking, perhaps I would have turned out better at this task. I love them both for all they've given me.

This book is based on the author's Ph.D. dissertation composed at Tel Aviv University's Cohn Institute for the History and Philosophy of Science and Ideas. A shortened version of the second chapter appeared in 2008 as a paper in Perspectives on Science, 16(2), 196–230.

Methodological Introduction

1. The components of this project

This is not an analytic project. I will not state a question, analyse it down to its constitutive conceptual elements, and attempt to derive a solution. This is a synthetic project. I will cut-and-paste patches of texts, and attempt to sew them together so as to force them into communication. Communication in this last sentence is not about the exchange of information. It has to do with **different or remote places communicating with each other by means of a passage or opening**. I will attempt to conjure communication as **a tremor [*ébranlement*], a shock, a displacement of force** (Derrida 1988a, 1).[1]

[1] As a synthetic project that depends on French critical tradition, this work replies on a heavy use of quotations. Quoted text here is not simply mentioned as object to be commented on, it is also used as quoted. This double gesture deserves a mark more prominent than the timid '...'. I therefore render quotations in boldface. Everything in boldface, except for headings, is quoted text, and all text that's intended to be read as quoted is in boldface. Italics inside quotations are always in the source text. I allowed myself to adapt spelling to UK standard, and made occasional minor punctuation changes to render the reading smoother. In a few places my quotations are presented in a sequence that does not correspond to their original order, and in one place (one only!) I allowed myself to indulge in the Talmudic liberty of quoting a text without a preceding **not**. In all such manipulated quotations the changes I allowed myself maintain the intention of the original message, as far as I can tell. Some of the other quotes, however, which are not manipulated formally, do subvert the 'author's intended meaning' to the extent that I understand this term. This is not simply the cost, but to an extent the

The textual components that I will attempt to bring into communication are post-structural semiotic theory and a couple of logico-mathematical texts. By 'logico-mathematical' I refer to Gödel's proof of his first incompleteness theorem. I read the proof in two versions: van Heijenoort's 1967 translation of the original paper from 1931, and the 1965 published notes of the 1934 Princeton lectures. Both versions were approved and revised by Gödel himself. References to these texts will be denoted by (1931) and (1934) respectively, and page numbers will refer to the first volume of Gödel (1986–2003). I focus almost exclusively on the proofs of the first incompleteness theorem in these texts, because they provide me with more than enough to work with. I do, however, occasionally refer to parts of the texts concerned with the second incompleteness theorem and with the construction of an arithmetic undecidable proposition.

I do not assume a familiarity with Gödel's argument, and a deep mathematical understanding of the entire argument is not necessary in order to follow this essay. Acquaintance with the elements of formal logic (propositional and predicate calculus) is, however, assumed. At some points (such as the section *The object: notion* in chapter 1) the discussion may get technical; I made an effort to contain such occurrences, and keep the text as accessible as possible to non-logicians without giving up a hands-on concrete analysis.

I use the term 'post-structural semiotics' to refer to French semiotics and discourse analysis developed between 1967 and 1974 by five prominent so-called '68 thinkers' (Foucault, Barthes, Derrida, Kristeva and Deleuze). From some of these thinkers I will borrow only guiding questions and methodologies, from others I will quote entire theoretic complexes.

I will not provide here a presentation of the theoretical edifice that I integrate into my text. Such presentation will be provided ad-hoc, as we go along. To attempt a concise introduction to some of the relevant thinkers would be either disrespectful to their distributed and non-stationary theoretic endeavours, or simply beyond the capacity of contemporary research,

benefit of forcing communication. Where I omit parts of a quotation (which happens quite often) I include a non boldface ellipsis or other unbolded text. When I quote words or small fragments from previous quotations I may or may not use boldface, and usually omit the reference.

which does not yet have the required perspective for such summaries. The textual patching of theory into my text, I believe, is the best presentation I can provide. I warn the reader, however, that this essay does assume openness to post-structural ways of thinking and writing.

The project that I aim for appears, at the very least, to go against the grain. Post-structural theory describes that which is unstable in language, that which is originally paradoxical, that which resists formalisation, that which is different-in-itself, that which is iterable beyond intention and context. Logico-mathematical texts, on the other hand, are supposed to be that stable rock to which science is anchored. 'Two is two is two' in an apparently much more undeniable and reassuring manner than even 'a murder is a murder is a murder'. Where it comes to murder, there may be attenuating circumstances; where 2 is concerned, some thinkers would claim, there need not be any circumstances at all.[2]

But a description of mathematics as the antonym of post-structural thinking is not precise. Some of the thinkers guiding this essay have alluded to Gödel's incompleteness theorem at one time or another. In fact, mathematics, together with poetry, held a distinguished position in the constitution of post-structural semiotics. The attention mathematics received was due to its non-phonetic script and its unique position with respect to signification. **The effective progress of mathematical notation thus goes along with the deconstruction of metaphysics, with the profound renewal of mathematics itself, and the concept of science for which mathematics has always been the model** (Derrida 1981, 35). Kristeva and Derrida mention Gödel while analysing Mallarmé and Philippe Sollers, and Deleuze quotes Russell and Whitehead while discussing Lewis Carroll's 'nonsense'. But as post-structural analysis has drifted further into literature, art, social studies, law and politics, it kept drifting away from mathematics, and communication between the mathematical inspiration and the post-structural practice has been dwindling away.[3]

[2]In the words of younger Husserl, **I see that, wherever is talk of the proposition or truth that π is a transcendental number, there is nothing I have less in mind than an individual experience, or a feature of an individual experience of any person** (Husserl 1970a, 300–301).

[3]For the relations between mathematics and post-structural thinking see Duffy (2004)

Post-structural thought used to be intimately related to modern poetry for a reason. As Kristeva puts it, **paragrammatism** (which I (mis)appropriate here as a generic term for post-structurally theorised semiotic effects) **being easier to describe at the level of poetic discourse, should be seized by semiotics first of all there, before exposing it in relation to all thought productivity** (Kristeva 1969, 176). Today, over 40 years later, it is time to attempt a forcing of post-structural thinking and that which has become its polar antonym, mathematics, into communication. In this revived effort I no longer seek to reposition mathematics as a source of theoretic inspiration. I wish to place it in the position of the analysand, a legitimate partner for post-structural semiotics and discourse analysis in the process of transference and counter-transference.

2. This project in contemporary academic context

The approach taken in this book is related to the what Philip Kitcher called the *maverick* trend in the study of mathematics. This tradition turns away from a foundational quest for the fortification of mathematics, and proposes a social and textual analysis of mathematics as a human activity. However, even within the maverick stream of thought, post-structural and semiotic approaches are rather rare (although not unprecedented, as witnessed by some papers in Ernest (1994)).

Among more tenured approaches, the closest ally of the maverick trend is Wittgenstein's philosophy of mathematics. For the later Wittgenstein mathematics is comprised of systems of rules, which are connected to each other by other rules. These rules are not arbitrary in that they are pragmatically and psychologically constrained; however, Wittgenstein refuses to acknowledge the derivability of such rules from any unified system, regardless of whether this system is formal, empirical, transcendental or platonic (this reading of Wittgenstein is substantiated by the quotations on page 136). Of course, this Wittgenstein is not unrelated to the Wittgenstein of

(around Deleuze), Charraud (1977) (concerning Lacan) and Tasić (2001), Ernest (1994) and Plotnitsky (2002) for general discussions.

the analytic tradition, who seeks to cure philosophical problems by setting apart different uses of words in different language games. In fact, in the context of Gödel's theorem itself, Wittgenstein sought to set apart and distinguish the different language games played with the word 'true' (Wittgenstein 1978, 118–122, esp. §8). But this is not the approach I take in this essay. Here I insist on the way that *the* language game depends on unstable and permeable boundaries between the different language games that it encompasses. Rather than a source of problems, I show that such unstable permeabilities are positive, constitutive forces for mathematical semiosis.

Another approach that's related to the maverick tradition is Husserl's phenomenology of mathematics and its interpretation by Derrida (1989). But while this approach attempts to account for the transition from intuitive perception to a rigorous axiomatic geometry, here we're concerned with semiotic processes that occur within rigorous mathematical production, and yet are not subject to axiomatised formalities.

Several contemporary authors should be mentioned to put this book in the present academic context. The first is Brian Rotman, who pioneered a novel semiotic approach to mathematics (Rotman 1993). His division of the mathematical sign-manipulating agency into a contextualised person, a context free subject, and a mechanical agent is critically engaged with in the first chapter of this book. I show that mathematical meaning requires forms of temporality and agency that cross, undermine and suspend the barriers between the above three aspects of the mathematical enunciative position.

The second author who should be mentioned here is Eric Livingston, whose early work, *The Ethnomethodological Foundations of Mathematics* (Livingston 1985) provides a detailed and careful analysis of the practice of reading and proving Gödel's theorem. Livingston rejects the option of relegating mathematical rigour and validity to a deferred transformation of the 'everyday language' proof into a formal reconstruction. The validity of the proof, according to Livingston, is anchored to the construction of mathematical practices and the organisation of these practices into **a structure of practices of proving, identifiably, just that theorem**. For Livingston, therefore, mathematical validity is an issue **in the pro-**

duction of social order (Livingston 1985, 17).[4] Yehuda Rav's work on the semantic aspects of mathematical work (Rav 2007) provides a more 'freestyle' version of related positions. My focus in this essay, however, is not the production of structures of validity, but rather the deconstructed proliferation and shifting of meaning. I demonstrate that mathematical practices of iteration and substitution prevent syntactic order from tying symbols to fixed meanings and uses, and that the construction of mathematical meaning, rather than being restricted to specialised mathematical and logical contexts, depends on more generic linguistic semiotic processes.

As the main focus of this project is the plurality of mathematical texts, one must acknowledge here the work of Claude Rosental concerning the plural aspects of classroom logic (Rosental 2008), and the textual analysis of multiple functions and **useful ambiguities** of mathematical signs in Lefebvre (2002) and Grosholz (2007). My own analyses of various other mathematical case studies using various theoretical perspectives, but always concentrating on the plurality of mathematical texts, can be found in Wagner (2009a,b, forthcoming).

Like readings proposed by the maverick tradition, my reading of Gödel's proof is conducted 'in context', but unlike most maverick readings, not always in the historical context in which the texts were written. Following hermeneutic approaches such as that of Gadamer (1975), the context of this project is the interaction between an old text and a contemporary reader. The texts mean something to me as I am reading, they confront me with

[4]The referee of a paper based on part of this book made the following objection concerning my level of analysis: **In case my complaint about remaining at the rhetorical level is not clear, let me point out that the standard epsilon-delta way of writing about limits, which has to do with the succession of quantifiers for all epsilon there exists a delta ... . This is sometimes written as a mock dialogue. If you choose epsilon, then I can choose delta ... . That does not turn the argument into such a dialogue. If one were to analyse such a dialogue, one would have to reconstruct it in terms of quantifiers to see the structure of the argument**. Like Livingston's, my stance in this paper is diametrically opposite. I am concerned here with mathematical practice, not with formal reconstruction. mathematics can be practiced without formal reconstructions, but has never historically existed without surface dialogues. I do not mean to dismiss formal reconstructions. I do object to the dictum that one **would have to reconstruct it in terms of quantifiers** to analyse the successful functioning of mathematics.

new horizons, and these effects of meaning are the stuff that I must account for. To get there I do, at some points, accept methodological restrictions of contextual reading. These methodological restrictions allow to bring up effects which, according to ideologies supportive of such methodologies, arise from the text without being imposed upon it. But since my project is not limited to historic interpretation, and does not wish to reconstruct Gödel's position, I hardly ever compare the translated text with the German one, and consider both Gödel's first paper and the subsequent lecture notes as perfectly valid and equally relevant sources to work with. My project is about communicating texts, not intentions.

3. The purpose of this project

Subject to its own claims on language, post-structural theory is well aware of its own instability, its self-subversion, and its dependence on — as well as interaction with — its objects of inquiry. Post-structural semiotics is therefore not an established set of tools to be applied to objectified texts. I will not pretend that my application of post-structural semiotic theory to the logico-mathematical is an experiment or case study in any scientific sense; rather it is a self-reflective performance of a scholarly rite. In this essay I will perform a post-structural reading with a logico-mathematical text. I will force the post-structural and logico-mathematical into communication.

The product of this project is not confined to a better understanding of Gödel's text or of post-structural semiotics. My primary question to texts I quote is hardly ever 'what do you say?'. Rather, my questions to quoted texts are usually of the form 'how do you work?' and 'what can I do with you?'. A response to these questions cannot pretend to bear the often neutral overtone of a term such as 'understanding'. Answering the question 'how do you work?' requires an intervention, namely the application of reading-tools, which necessarily transform the quoted text by breeding it with the hermeneutic and analytic stances of the reader and her or his tools. Such transformative intervention is precisely how texts are made to respond to the question 'what can I do with you?', and become partners to conversation.

As the goal of this project is to produce the communication articulated above, my textual quilting will prove a successful endeavour to the extent that it is cut and integrated into other texts, to the extent that it is quoted, reiterated and repeated, to the extent that, once worn out by the fittings and bodies which it might serve to cover, it continues to provide manufacturers of text with raw material for their production. A more optimistic writer might even dare to hope that while flowing as current through circuits of textual proliferation, this text might be able to induce a field of force acting on human practices that exceed the editing of texts, such as education and academic prioritising.

I hope that my textual patch-work will prove useful for some readers of the text, be they mathematicians, philosophers or others. I hope that my textual suturing will open the quoted texts to further, not necessarily anticipated uses. These hopes, however, have a rather problematic interaction with notions of truth. Parts of this essay are designed to form a legitimate analytic reading, as far as I understand what distinguishes such readings — after all, even if one objects to their contemporary privileged status, one can admire their productivity. But I play several language and truth games with changing degrees of irony and commitment. I am indeed committed — for a few pages or throughout this essay — to some determinations of truth. But telling the truth is not a primary business of this project (for example, the psychoanalytic edifice of chapter 3 is not supposed to be a true description of fact; it is used as a tool for thinking language and bodies together).

One important use I have in mind for this project is ethical: a reevaluation of the authority invested in mathematical texts. The term ethics is meant here as concern with taking responsibility for a text. Who is responsible for a text? Is it the author? the editor, publisher and distributor? the voices which express their statements in the text (statements which may run contrary to those of the author, editor, publisher and distributor)? Is it the reader who interprets the text and, perhaps, replicates its statements? Is it the social or formal order of language that enables the writing of the text? Or is it simply truth and reality, to the extent that the text states the real truth? Given this complex web of responsibilities for a text, to what extent can we invest a text with authority — an authoritative

text? I believe that the picture I draw in this book has substantial implications, if concern with these questions afflicts mathematical texts. I hope that the textual quilt I sew together will have an impact on the derivation of authority from mathematics' supposedly privileged access to truth and meaning, and on the authoritative privileges bestowed upon mathematics in contemporary academic discourse due to that supposition. But this issue of authority is not the same as that of validity. I have no qualms about the logico-mathematical validity of Gödel's proof.

To summarise:

- This is not a historical essay. It does not frame Gödel's proof in terms of chains or flows of practices or ideas.

- This is not a mathematical essay. It does not explain the proof, nor does it cast doubts on its mathematical validity.

- This essay analyses Gödel's proof as an example of mathematical practices. 'Mathematical practices' stand here for reading (understanding) and writing (producing) mathematical texts.

- I believe that there is no deeper essence to mathematics beyond its practices of reading and writing (in an extended sense, which includes contemplation, application and transcription). This essay, however, does not seek to rule out alternative beliefs.

- Articulating the features that distinguish mathematical practices from other textual practices is an interesting task. This essay, however, touches upon this task only marginally.

- This essay shows that reading and writing Gödel's texts depend on the paradoxical and unstable semiotic processes that post-structural theoreticians diagnosed in 'softer' textual practices.

- As mathematical practices enjoy the paradoxical and unstable semiotic processes operative in 'softer' textual practices, it is difficult to make an *a priori* claim for mathematics' privileged authority or distinguished access to truth. Claims that mathematical practices have a privileged authority or a distinguished access to truth had better be

established *a-posteriori*, in terms of the implications of mathematics, its results and applications.

4. The chapters

The first chapter is devoted to a discursive analysis of Gödel's texts guided by methodological devices articulated in Foucault's *Archaeology of Knowledge* (Foucault 1972) and in Barthes' *S/Z* (Barthes 1974). My first attempt included a bit-by-bit analysis of the introduction to the 1931 text using the methodology that Barthes presented in *S/Z*. The process was very productive, but presenting the results according to Barthes' formalism appeared to be tedious and rhetorically difficult to sustain. I therefore imported the results of this analysis into a slightly more 'archaeological' analysis of the text.

Carrying such an archaeological analysis of Gödel's texts, however, is not without its problems. Foucault explicitly determines his archaeology to refer to much larger bodies of texts. Indeed, he explains, **I do not wish to take as an object of analysis the conceptual architecture of an isolated text, an individual *oeuvre*, or a science at a particular moment in time. One stands back in relation to this manifest set of concepts; and one tries to determine according to what schemata (of series, simultaneous groupings, linear or reciprocal modification) the statements may be linked to one another in a type of discourse** (Foucault 1972, 60). Nevertheless, the imposing of archaeological leading questions on Gödel's texts, in conjunction with the conclusions carried over from the Barthesian analysis, provided a stimulating point of departure for my discussion.

In this chapter I quote a lot from Gödel, and less from Barthes and Foucault. The main reason for this practice is that Foucault's and Barthes' theories are still presented as theories in the traditional sense, namely as tools that efface themselves as they are being used to produce an objective analysis. Both Foucault's *Archaeology of Knowledge* and Barthes' *S/Z* stand at a liminal point between structuralism and post-structuralism, and therefore allow gradual transition into post-structural reading. The analyses are structural in that they articulate units and review their combinatorial

interrelations; they exceed structuralism in that the articulation of units is opened to extra-textual intervention, in that they do not presume to locate syntactical arrays that are independent of the reader, and in that the analysis is not governed by a belief in universal combinatorial determinations of possible results.[5]

One should not expect in this chapter an exhaustive appropriation of Barthes' and Foucault's theoretic edifices. In many ways large parts of this chapter can be subsumed under more traditional hermeneutic frameworks, such as Gadamer's decree that **the text, whether law or gospel, if it is to be understood properly, i.e. according to the claim it makes, must be understood at every moment, in every particular situation, in a new and different way. Understanding here is always application** (Gadamer 1975, 275).

The second and third chapters include a synthesis of the mathematical texts with Derrida's and Deleuze's semiotic constructs respectively. Substantial references to the work of Kristeva cross through both chapters. From Derrida I quote mainly *Dissemination* (Derrida 1993), *Of Grammatology* (Derrida 1976), the paper *Signature, Event, Context* from *Limited Inc.* (Derrida 1988a), and a few other texts. From Deleuze I quote *The Logic of Sense* (Deleuze 1990) and *Difference and Repetition* (Deleuze 1994), and from Kristeva the non-translated *Semeiotike* (Kristeva 1969) and the *Revolution in Poetic Language* (Kristeva 1984) (the translations from the former are my own).

As I wrote above, the introduction of the relevant theoretical elements will be threaded into the discussion of Gödel's proof. The question 'how does the mathematical text make sense?' is used as catalyst to instigate Gödel's texts and post-structural semiotics into interaction.

In these two chapters I refer directly and explicitly to Gödel's texts, but do not quote from them as often as in the previous chapter. I preferred this slightly looser form of interaction in order to allow my text enough motility and independence so as to be more readily applicable to other mathematical texts as well. I believe that semiotic analysis can only be performed concretely on each given text, but I attempted to open up my

[5] These characterisations of structuralism are more a caricature than a faithful description, but I do not wish to attempt a precise historic articulation here.

concrete analysis so that it can be reappropriated concretely to other texts as well. Derrida, Deleuze and Kristeva, on the other hand, are quoted quite intensively. This is done not only in order to introduce the relevant theoretical constructs, but also in order to force these theoretical constructs to expand their horizon so as to comment on Gödel's texts, with which they were not actually concerned.

The question of similarity and difference between Deleuze and Derrida is starting to get some attention (e.g. Patton & Protevi 2003). I believe that it is much more interesting to confront these two thinkers via the mediation of third elements than by seeking to list elements of congruence and divergence between their positions as such (the latter approach is almost disrespectful of their achievements). I hope that the third and fourth chapters of this essay serve to deepen our insight into how they repeat, *differenciate* and *iter*ate each other.

The appendix concluding this book is a rather awkward attempt to confront Gödel's proof with narrative texts and the notion of structure. This is a useful preliminary exercise, if we are to impose on a mathematical text post-structural semiotics — an approach that emerged from structuralism and from literary criticism.

Mathematics is often presented as the structure of something (reason, language, the universe). But to bring it closer to literature, I needed to take this way of thinking one step further. I (ab)use Lévi-Strauss' conceptual framework to attempt an assignment of Gödel's text into the position of structure with respect to a made-up mythical text. Readers, who object to reading a text as something that it obviously is not, are referred to Borges' *Pierre Menard, Author of the Quixote* (Borges 1962, 45–55). I cannot improve upon his apologia for the method of anachronistic attribution.

The failure of my fake reading of Gödel's proof as the structural analysis of a lost myth, which I set as my task to retrace, is grotesque but instructive. It brings out the limitations of a 'common structure' hypothesis (common to myth and science, or to human 'untamed thinking' in general). It reaffirms the specificities of literary and mathematical texts, and explicates their relative discrepancies. The texts distributed across these discrepancies and specificities have, nevertheless, this much in common: they all deserve the benefit of a reading that respects their specificities and

discrepancies, and that does not assume that they must be subordinated to any particular given text or phenomena.

There is also another excuse for this awkward first chapter. I felt that it would be counter-productive to provide yet another layman account of Gödel's argument; there are quite a few such accounts, some of which are very good, (e.g. Nagel & Newman 1958), and I am unlikely to do any better. The mythologising of the proof might make it a little more accessible to people with little logical background, without actually presuming to explain the proof.

This exercise has some merit, but is also somewhat detached from the rest of this project. It served me well, but I am not sure that it will perform the same service for all readers. I therefore advise readers familiar with Gödel's argument and readers with analytic tendencies to skip this chapter, or perhaps read it as an appendix. Other readers may want to read it first.

Analytic Introduction

1. Meaning

This is not an analytic project. But I see no reason to prevent it from being read as such. I see no reason not to read this project as attempting, through a careful analysis of a distinguished case study, a new attempt at articulating the question 'how does mathematical meaning work?'

I will not confront this question by articulating minimal elements or determinations of meaning, and then describing how these minimal elements combine to generate the global meaning of an entire mathematical text. To the question 'how does meaning operate?' I reply, inspired by the thinkers whom I quote: *Meaning is an operator which produces difference across repetition.*

What kind of operator? A linguistic operator for those who took the linguistic turn, an experiential operator for the phenomenologically inclined, a self-defeating transcendental operator for metaphysicians,[1] and an embodied performative operator for theoreticians whose positions involve the prefix 'post' (structural, modern, feminist, colonialist...).

I do not presume to exhaust with the above formula the entire scope of the operation of meaning. In addition to producing difference across repetition (manifest in such statements as 'this *same* term is employed with *different* meanings') it obviously produces repetition across difference ('these

[1] Self defeating because it undermines the structure that metaphysicians usually require transcendental objects to hold on to.

two *different* expressions mean the *same* thing'). Furthermore, bringing up meaning obviously has various other effects, such as the transcription of a given statement in terms of intention, extension, truth conditions, truth value, assertibility conditions, grammatic or otherwise formal structure, stimulus-response arrays, phenomenological reduction, ideal source and quotability, to name but a few. But I do maintain that instilling difference across repetition is as fundamental to the operation of meaning as any of these other terms; 'as fundamental' and not 'more fundamental', because imagining a linear chain of foundations which goes all the way down to a primary cause is precisely the image I do not have in mind (or, rather, have in mind under a stroke of erasure, repeated differently), when I define meaning as an operator that instils difference across repetition. The fundamentality of my definition is demonstrated by articulating the classification of phrases according to intention, extension, etc. as marking different phrases as somehow repeating each other, or various repetitions of a phrase as carrying different meanings. Still, it would be more just to say that my italicised definition above is not a definition of meaning, but the articulation of an aspect of meaning, which is as definitive as any of its other aspects.

Let's be more explicit. In *How to Do Things with Words* Austin coins a disturbing example. **He said to me 'shoot her!' meaning by 'shoot' shoot and referring by 'her' to *her*** (Austin 1962, 101). This example is disturbing, because it convincingly demonstrates, but fails to explain, in what way the sterile repetition of **shoot** and **her** (with and without quotation marks, in a medium or italic font) serves to explain or clarify anything. It is important to note that Austin's intervention instils difference across repetition. **shoot** and **'shoot'**, **her** and ***her*** deploy different functions, different grammars, uses, intentions, extensions, references, affects and idealities because, if they had been *exactly the same*, it would have served no purpose to repeat them.[2] The fact of repetition entails that the elements **shoot** and **her** must decompose into smaller constitutive elements or discharge a sort of excess: the element 'itself' and the element's 'ghost', *use* and *mention*, the text animated with meaning as opposed to

[2]Is this even possible? Can something repeat something *else*, which is exactly the *same*, without collapsing into unity?

the quoted text. Note, however, that none of Austin's locutions, illocutions and perlocutions in his written discussion of **'shoot her!'** have ever been 'actually' used (I hope), in the sense that they have never given rise to a single actual shot.[3]

This procedure of effectuating repetition to produce difference is not the invention of 20^{th} century analytic philosophy. This procedure is, among other things, a fundamental technique of the Jewish *Talmud*, the interpretation of the codices deriving laws from the old testament. In the Talmud this procedure is referred to by the Aramaic term *pshita*, which literally means simple or obvious. When the interpreter encounters a repetition of an already stated fact or of what is already commonly known, he makes the assumption that the apparent repetition is justified by a new and original statement which is folded within it. This novel meaning is excavated by the interpreter through instilling difference across the repetition. *Interpretation abhors repetition.*

But there's nothing particularly Jewish in this manoeuvre. The most exemplary analytic philosopher ever (who is not recognised as such only because of the trifling fact that he was confined not by 20^{th} century western rationalism, but by 11^{th} century Catholic dogma), Anselm of Canterbury, writes: **The rational mind, then, when it conceives of itself in thought, has with itself its image born of itself that is, its thought in its likeness, as if formed from its impression, although it cannot, except in thought alone, separate itself from its image, which image is its word**. We see here that thought alone can instil a difference between the thought image that repeats the thinking mind. And in relation not to just any rational mind, but to that rational mind, a more excellent than which is inconceivable, he writes: **But although this is true, yet it is most remarkably clear that neither he, whose is the Word, can be his own Word, nor can the Word be he, whose Word it is, although in so far as regards either what they are substantially, or what relation they bear to the created world, they ever preserve an indivisible unity. But in respect of the fact that he does**

[3]Likewise, it seems to me that 'Could you pass me the salt?' appears much more often in classroom discussions of linguistic pragmatics than around dinner tables. It has been suggested, however, that perhaps I do not dine in very good company.

not derive existence from that Word, but that Word from him, they admit an ineffable plurality, ineffable, certainly, for although necessity requires that they be two, it can in no wise be explained why they are two (Anselm 1903, chapters 33, 38). Even when no difference can be claimed concerning substance and relation to the world, the difference between *expressing* and *expressed* is instilled across the repeated substance and identical relation to the created world. The Father and the Word, the Father and the Son, are forced apart, even though **it can in no wise be explained why they are two**. Shoot and 'shoot', *her* and 'her'.

But difference across repetition doesn't end there, with *signifier* and *signified*, or with *mention* and *use*. A simple exercise of countertext (replacing portions of text by other text in order to investigate their differences) will demonstrate the proliferation of difference across repetition. Rather than **meaning by 'shoot' shoot**, let's contemplate 'meaning by "meaning" meaning'; rather than **referring by 'her' to *her*** — 'referring by "by" to *by*'. The first 'meaning' and 'by' of our object statements are *actually used*.[4] The second and third iterates of 'meaning' and 'by', on the other hand, act respectively as the signifier and the signified of the *mentioned* 'meaning' and of the *mentioned* 'by'.

If one is not quite convinced that there are here three separate linguistic roles, then, again, an exercise of countertext can avail. Replace one of the occurrences of 'meaning' by 'eating' in the statement under analysis. The results are: 'eating by "meaning" meaning', which sounds senseless (in a 'serious' context); 'meaning by "eating" meaning', which would make sense, if we were describing a secret code or a slip of the tongue; and finally 'meaning by "meaning" eating', which would make sense under the same hypothesised circumstances, but demonstrates the asymmetry between the two aspects of the mentioned 'meaning'.

The game obviously does not end here. Consider now 'meaning by "meaning by 'meaning' meaning" meaning by "meaning" meaning'... This line of thinking leads to at least two alternatives (which do not presume to exhaust the scope of possibilities in dealing with this chain of 'meaning's): either assign a distinct role to each 'meaning', or, refusing to acknowledge

[4] More precisely, the first 'meaning' and 'by' of our object statements are presented as actually used, since no order is actually given, and no shot is taking place.

a dissection of linguistic meaning into such contrived and esoteric elements, discard the entire sequence as meaningless; either produce difference across repetition, or refuse to assign meaning to each of the repeated elements. Either way, we see that assigning meaning to text is bound here with treating repetitions differently.

There are, of course, other ways of confronting this long chain of 'meaning's: refuse to assign meaning to the elements, but acknowledge the meaning of the whole sentence. 'What does it mean? It means that you shouldn't iterate too much!' Here the constitutive questions would be 'What does the entire sequence repeat?' (to which we could answer: 'the grammatic operation of substitution of a phrase into a formula') and 'How is it different from that which it repeats?' (to which we could answer: 'in that we exclude it from the realm of making sense'). With this approach too, we are still producing difference across repetition.

In the discussion above are implicit two conceptions of meaning. The first conception of meaning can be described as 'structural', and we shall quote its succinct definition from Barthes. According to the earlier Barthes all semiotic terms **necessarily refer us to a relation between two relata** (Barthes 1968, 35). To manage such relations between relata in a manner that avoids collapse, one applies *segregation* and *censorship*. Segregation sets elements apart by restricting their positions and interactions (e.g. objects and predicates, where an instance of the latter may apply to an instance of the former, but not to itself, at least in first order logic). Censorship strikes off texts that threaten the stability of the system (e.g. Russell's choice to strike off 'the set of all sets that do not belong to themselves' from his system).

The second conception of meaning, which we will call 'post-structural', and the succinct formulation of which will be borrowed from Derrida, says that **iterability... structures the mark**. *Iter*, here **comes from *itara, other* in Sanskrit, and everything that follows can be read as the working out of the logic that ties repetition to alterity** (Derrida 1988a, 7). Here meaning entails productivity (of difference from repetition), mystery (repetition is known to produce difference, even when it is not yet known which difference, even when repetition produces nothing but the mystery of a difference-to-be-specified, as in the *pshita* or the divine and its

word) and dissemination (distinct elements of meaning collapse or multiply as we try to pin them down).

With this last conception in mind, searching for meaning is akin to a clumsy search for marbles in the dark. As one searches for marbles in the dark, one bumps into the marbles, thereby causing them to roll, bounce off each other, and change their locations. Similarly, as one searches for meanings of texts, one quotes them and uses their words, thereby changing their network of relative positions and significations. But unlike the clearly individuated marbles, where meanings are concerned, there is no single articulation into meaning units (words? phrases? assertions?), and no clear counterpart of the concept of 'location'.

From this point of view one can caricature the projects of Derrida, Kristeva and Deleuze respectively as: (1) establishing that Barthes' definition of sign keeps breaking down ('deconstructing') into **the logic that ties repetition to alterity**, (2) studying the operation of signs as material iterates, and (3) reconstructing a revised view of signification and meaning from the sign structurally determined as an 'iterable'. These projects will be confronted with Gödel's mathematical texts. My task in this project is to highlight within the mathematical text the aspect of meaning as production of difference across repetition, and to explore the effects of this operation.

2. Leading questions

This work is led by the intuition that mathematical language, like other forms of language, despite its peculiarities and particulars, enjoys, at its heart, the full complexity of language as a process, with its genesis, constitutive paradoxical forces, unbounded chains of referencing, and contingent strategic elaborations. Like language in general, logico-mathematical language is the admirable product of **a mighty genius of construction, who succeeds in piling an infinitely complicated dome of concepts upon an unstable foundation, and, as it were, on running water. Of course, in order to be supported by such a foundation, his construction must be like one constructed of spiders' webs: delicate enough to be carried along by the waves, strong enough**

not to be blown apart by every wind (Nietzsche 1979, 85).

It is important to note that my project does not aim to attack the logico-mathematical validity of Gödel's argument. Gödel's argument has been subject to extreme scrutiny, and has resisted many attempts to interject gaps within it and undermine its scope (for early reception see Dawson 1997, 75–77). My aim is not to undermine Gödel's text, but rather to study what makes it function successfully. It is also important to note that the scope of this essay does not include Gödel's position in a history of mathematical ideas, or a symptomatic reading of Gödel's text that will unveil a deeper psychological or philosophical encoded message.

My questions concern processes of semiosis: *what is it in the language of the text, which allows it to function for a mathematical reader*, be that reader a contemporary of Gödel or a contemporary of the reader of this text? To what extent *does the functioning of the text have anything to do with post-structural key notions* such as (non)-sense, difference and repetition, amnesia, hymen, castration, verisimilarity, transcription formations and modalities of enunciation, to name but a few? *Is mathematical language subject to the paradoxical motions, genetic processes, and strategic manoeuvres, which set 'natural' language in motion* according to post-structural theorisation?[5]

Let's give some examples, in order to be more explicit. Can mathematical language contain such problematic statements as Magritte's **Ceci n'est pas une pipe**, where **Ceci** may or may not refer to the drawn pipe, the drawing of the pipe, the word pipe, the signified of the word pipe, as well as to itself as a manifest subject, pulling itself by its own hair outside the swamp of designation? Can mathematical language contain the kinds of shifts of meaning that we're used to expect from poetry? Can we find in

[5] If the answer to these questions be positive, we should expect mathematical language to allow for another effect of 'natural' language: jokes. There are indeed many jokes about mathematicians, but that is not what we are after. I know at least one joke which is *in* mathematics. It is *in* mathematics both in the sense that it is written in strictly mathematical language, and in the sense that I know of at least two researchers who independently claimed to have written professional papers on this joke and its generalisations (Prof. Y. Rodity and the late Prof. S. Breuer from Tel-Aviv University). It is a *joke* in the sense that when told in an appropriate setting it makes people laugh out loud. Here is the joke: $\frac{\cancel{6}4}{1\cancel{6}} = 4$.

the mathematical text outbursts of embodied *desire*, double articulation, or a history of castration? Suppose all this is indeed the case, as I will attempt to demonstrate; does it, or does it not establish within mathematics a mathematical unconscious and a mathematical will to power, which will make the reservation 'within mathematics' redundant?

For the purpose of making my questions concrete I include a teaser of the analysis of a specific instance of repetition central to Gödel's text (the full analysis will take place in chapter 2; here I cannot provide a decent exposition of context and details, but a summary account of Gödel's argument is provided in the penultimate section of this introduction). But bear in mind that this is only a teaser, not an attempt to make an argument.

The first step in Gödel's argument is to enumerate all formulas in a certain formal language. Each formula is assigned a unique number, which it shares with no other, and which serves as a sort of name for the formula. Then, given numbers a and b, Gödel denotes by $S(z_a, z_b)$ the number that corresponds to the formula obtained through to the following recipe:

- Take the formula whose number is a
- Substitute the number b for some specified free variable inside that formula

If we ignore a reading restricted to a purely formal layer of the text (a privilege which, of course, I will not claim without justification outside this introduction), we see that the first number, a, refers to a formula, whereas the second, b, refers to a number.

Now, if we take, as Gödel does, a repetition of the form $S(z_p, z_p)$, then the *same* p has *different* referents (formula and number). This alone is not alarming at all, and does not require post-structural contemplation. The word 'watch', for example, can play two different noun roles: sentinel and clock. When we say 'the watch strolled by the watch', the *same* word has *different* referents. We can even play around with referent assignment: 'the watch's watch' could mean either the sentinel's clock, or the clock's sentinel. All we have here is a simple case of polysemy.

But, as I will show in chapter 2, this is not quite the case with Gödel's $S(z_p, z_p)$. Indeed, in this repetition both p's arise by substituting a *single*

number for the *same* free variable in a certain tailor-made formula. A single p adopts two *different* roles by means of a most pedantically regulated *repetition*. We shall ask about this instance of repetition whether it repeats a given concept, or whether it is a motion of decontextualisation that produces, within repetition, difference and a residual meaning. The question is, in other words, does the emerging *difference* already exist a-priori, like the *difference* between the two distinct roles of 'watch' in 'The watch's watch', or does this form of *repetition* produce an a-posteriori residual *difference*? Does this *repetition* conform to a stable system of distinctions, or does it have the transformative, self-refuting role reminiscent of Mark Anthony's repetition **for Brutus is an honourable man**?

The question further extends to the very place into which p is substituted. Is it a well-confined vacuum, or is it rather like the ambivalent place of a misplaced library book, a book which, according to Lacan, **manque à sa place** (Lacan 1966, 25) — missing in/from its place, a place occupied (the book is where it's at, in its place), and yet empty (the book is not where it's supposed to be, not in its place)? Does p, like the library book, or an oyster, participate in producing and carrying its place? Could we perhaps relate to this awkward place Foucault's analysis of Velasquez' *Las Meninas* in *The Order of Things*? **Here, the action of representation consists in bringing these two forms of invisibility into the place of the other, in an unstable superimposition — and in rendering them both at the same moment at the other extremity of the picture — at that pole which is the very height of its representation: that of a reflected depth in the far recess of the painting's depth. The mirror provides a metathesis of visibility, which affects both** (qui entame à la fois) **the space represented in the picture and its nature of representation; it allows us to see, in the centre of the canvas, that which in the painting is of necessity doubly invisible** (Foucault 1973, 8).

Introduction to Gödel's Argument

1. Logicism, intuitionism, formalism, realism and Kurt Gödel

The philosophical juncture at which Gödel operates and his own position are not the concern of this book, but a basic acquaintance would nevertheless be useful for the discussion. The three main paradigms for providing a foundation for mathematics at the time when Gödel wrote his proof were Logicism, Intuitionism, and Formalism. The sketches of these three positions are quoted from Gödel's own review of Carnap's, Heyting's and von Neumann's respective presentations (Gödel 1986–2003, Vol. I, 243–249). Gödel explains that **the thesis of logicism is twofold, namely, (1) that all mathematical *notions* are reducible to logical ones through explicit definitions and (2) that all mathematical *theorems* are deductively derivable from the principles of logic**. Logicism is, then, a full reduction of mathematics to the principles of logic.

For the intuitionist, on the other hand, **mathematics is a natural function of the intellect, a product of the human spirit, and** he **therefore grants no objective existence, independent of thought, to mathematical entities. This conception — that mathematical objects exist only insofar as they can actually be comprehended by human thought — leads to a rejection of pure existence proofs, as well as of the principle of the excluded middle in all cases in**

which a decision among the alternatives cannot actually be made. The intuitionist, then, requires a constructive mental image, graspable by the finite human mind, for every concept and every method of inference to be allowed in mathematics.

The formalist position, in relation to the two preceding positions, tries to have the cake and eat it too. **The formalist's goal in providing a foundation for mathematics is ... to vindicate classical mathematics while taking into account the doubts raised by intuitionists**. To do so, **he directs his attention to the fact that** infinitary and non-constructive **modes of inference attain a thoroughly finitary meaning if they are viewed, not as methods of conceptual thinking, but as a procedure for deriving formulas according to certain conventions, since the formulas and the operations carried out with them do, after all, have a finitary character**. The validity of these finitely graspable and verifiable formula games is not to be measured against some intended meaning, which may involve infinite and non constructive manoeuvres; rather it is measured by their consistency with the realm of finite mathematics. Any infinite or non-constructive speak will be acceptable, as long as its deductions can be finitely verified according to syntactic rules, and as long as it does not force contradictions into finite and constructible mathematics.

Gödel does not quite fit into either of these frameworks. He adopts Hilbert's formalism, and already in his 1929 thesis (Gödel 1986–2003, Vol. I, 60–101) establishes himself as a virtuoso of this newly established approach to mathematics. He nevertheless refuses to accept Hilbert's horizon of reducing mathematics to formal procedures, and rejects formalism as *the* basis for *all* mathematical knowledge. Indeed, Gödel's incompleteness theorems undermine both the second component of the logicist agenda, and the hope for a consistency proof that would put formalism on firm ground.

All that does not mean that Gödel is an intuitionist. He is willing to handicap himself by the intuitionists' reductive arsenal of proof techniques in order for his results to be universally accepted, but refuses to acknowledge intuitionism as a sole guarantee for mathematical rigour. Despite Gödel's ability to conform to the standards of formalism and intuitionism, Solomon Feferman defines him in his introduction to Gödel's col-

lected works as a 'Realist' or 'Platonist'. These terms stand for Gödel's counter-intuitionist belief that mathematical concepts have a genuine predetermined existence, which bestows a unique a-priori truth value upon all mathematical statements. Feferman states that in Gödel's view, not only formulas and logic can serve as a basis for mathematics, but that also **mathematical intuition** (not to be confused with intuitionism) **can be a source of genuine mathematical knowledge. This intuition can be cultivated through deep study of a subject, and one can thus be led to accept new basic statements as axioms** (Gödel 1986–2003, Vol. I, 30–31). These claims are strongly supported by Gödel's paper *What is Cantor's Continuum Hypothesis* (Gödel 1986–2003, Vol. II, 254–270). To appreciate Gödel's position one should consider that while he had formally proved that a certain mathematical statement is consistent with axioms that he considered to be evident, he nevertheless conjectured that the very same statement was actually false.[1]

2. Gödel's argument in brief

I will conclude this introduction with a brief partial overview of Gödel's argument, which I hope will set this essay in its mathematical context.

Gödel's argument concerns a standard formal system (based on Russell and Whitehead's *Principia Mathematica*) with a fixed set of symbols for logical operators (such as conjunction, negation, universal quantifier, etc.), functions, constants and variables. It is crucial that the formal system can represent the universal quantifier ($\forall$, read 'for all'), a negation connective ($\neg$, read 'not'), and the natural numbers. Explicit and finitely verifiable syntactic criteria determine whether a given sequence of symbols is a legitimate formal expression, or in Gödel's terminology, a *formula*.[2] Finally, an explicit set of syntactic rules decides whether a sequence of formulas constitutes a proof.

[1] I refer here to the consistency of the Continuum Hypothesis with the ZF axioms proved in Gödel (1986–2003, Vol. II, 33–101) and objected to in Gödel (1986–2003, Vol. II, 254–270).

[2] *Formula* here should be thought of as a proposition or statement, rather than as a formula for computing or constructing something.

Gödel's argument proves that, unless the formal system is inconsistent,[3] there exists a formula in the language, such that neither this formula, nor its negation are provable. Such formulas are called *undecidable*. A formal system that has undecidable formulas is called *incomplete*. Succinctly, Gödel's first incompleteness theorem states that if the formal system is consistent, then it is incomplete. The scope of the argument was shown by Gödel to cover not just one specific formal system, but to include a wide variety of formal systems, which include all the 'mainstream' systems that can represent natural numbers.

The first component in the argument is a method for translating any finite sequence of the formal language's symbols into a number. This translation method will not be reviewed here (we review it in the second section of the appendix), but it is important to note its following properties:

1. No two symbol sequences correspond to the same number.

2. Given a symbol sequence, its number can be computed by a finite mechanisable procedure.

3. Given a number, the symbol sequence that corresponds to it can be computed by a finite mechanisable procedure.[4]

Note that this enumeration covers all symbol sequences, and includes, among others, those that make up legitimate formulas and legitimate proofs according to the system's syntactic rules.

The next component of the argument is to prove that various formal relations between formulas can be translated into arithmetic relations between the numbers corresponding to these formulas. 'Arithmetic relations' refer here to relations based on arithmetic operations that can be expressed by the limited vocabulary of Gödel's formal language. For instance, the relation 'The symbol sequence numbered x proves the formula numbered y' can be translated into an arithmetic relation between the numbers x and

[3]Inconsistency means that there exists a formula, such that both it and its negation are provable. However, there is a delicate reservation here that I postpone until the conclusion of the argument.

[4]Not all numbers need correspond to some sign-sequence, but that will not affect the argument.

y, and this arithmetic relation can, in turn, be expressed in the formal language. We will denote here this formal relation by $P(x, y)$. In fact, Gödel demonstrates that symbol sequence number x proves formula number y if and only if the relation $P(x, y)$ can be proved in the formal system; moreover, symbol sequence number x fails to prove formula number y if and only if the relation $\neg P(x, y)$ — the negation of $P(x, y)$ — can be proved in the formal system.

Via a clever construction Gödel produces a number g, such that the following formal sequence:

$$\forall x(\neg P(x, g))^{5}$$

is numbered g. Therefore g is the number of the formula that states that no number x corresponds to a proof of the formula numbered g; simply put, the formula numbered g states that the formula numbered g (itself) is unprovable. The negation of the formula numbered g would say, then, that the formula numbered g is provable.

The argument is now easy to recapture. First we shall show that, unless we have an inconsistency, formula number g cannot be proved.

- Suppose that the formula numbered g had a proof.
- The proof of the formula numbered g would then be a sign-sequence. Let its number be y.
- We get that the sign-sequence numbered y is a proof of the formula numbered g.
- According to the explanation above, this implies that we can prove $P(y, g)$.
- On the other hand, if we could prove the formula numbered g, namely $\forall x \neg P(x, g)$, we could also substitute y for x and conclude $\neg P(y, g)$.
- But the last two conclusions are inconsistent.

Now we turn to showing that the negation of formula number g cannot be proved.

[5]This reads: for every x, the symbol sequence numbered x does not prove the formula numbered g.

- Suppose we could prove the negation of the formula numbered g.
- This would mean that the formula numbered g would be provable.
- But we have just shown above that this would yield an inconsistency.

Note that this argument relied on a semantic move ('this would mean that...'), based on our interpretation of the formula numbered g. This is the so called *semantic argument*. If we work more rigorously we obtain the *syntactic argument*:

- Suppose we could prove the negation of the formula numbered g, namely the formula $\neg\forall x(\neg P(x,g))$.
- We already know from the argument above that the formula numbered g cannot be proved.
- Therefore, for every number x, the sign-sequence numbered x is not a proof of the formula numbered g.
- It follows that for every x we can prove $\neg P(x,g)$.
- This is inconsistent with the initial hypothesis.

But the last statement is not completely precise. We would get an explicit inconsistency if we proved in the penultimate step the statement $\forall x\neg P(x,g)$. But what we actually got is 'for every x we can prove $\neg P(x,g)$'. There is no formal inference which allows to deduce the former from the latter. To obtain the latter formally, we would require a 'template' into which we could substitute x and get a proof of $P(x,g)$; all we actually have is for each separate x a separate proof of $P(x,g)$.

The situation that we have obtained (for every x a proof of $P(x,g)$ and at the same time a proof of $\neg\forall x\neg P(x,g)$) is termed by Gödel ω-inconsistency, and his claim concerns this kind of inconsistency. Gödel proves that unless the formal system is ω-inconsistent, it must contain an undecidable formula. This result was later improved by Rosser (1936) to replace ω-inconsistency by plain inconsistency (provability of a formula and its negation).

Gödel concludes his argument with a move that cannot be validated in the formal system where the proof was set. What Gödel showed is that

the formula numbered g is unprovable. But, at the same time, the formula numbered g, as introduced above, claims that the formula numbered g is unprovable. Therefore, the formula numbered g makes a true claim. A lot of attention has been given to this manoeuvre and to its validity. In particular, to retain the consistency of this manoeuvre, the truth appealed to in this argument cannot be something provable in the formal system. This led Carnap and Tarski, followed by Gödel himself (in section 7 of the 1934 text), to construct hierarchical systems of languages, where the truth predicate of each language belongs to a higher language. We will touch upon these issues in this book, but will not focus on them.

There is one more delicate point. The argument as presented above corresponds to the 1934 version of the text. In the earlier 1931 version the place of meta-mathematical claims (dealing with symbol sequences, proofs, etc.) is taken by arithmetic claims (dealing directly with the numbers corresponding to sequences, proofs, etc.). For example, instead of claiming 'Formula number g is provable', the text makes an arithmetic claim about the number g. This coding makes the proof a little more tedious, but in stricter correspondence with a view of mathematics as dealing with numbers, rather than with such notions as proofs. On the other hand, this coding through numbers distances the claim from an explicit determination of undecidability. This tension will be reviewed in the second section of the first chapter of this book.

Chapter 1

Textual Formations

In this chapter we will analyse the subject position and transcription devices operated by Gödel's text. The product will not be a structured formation. We will hear a plurality of voices and observe the marks of reiterated reformulation.

The questions leading the analysis in this chapter are derived from Barthes' *S/Z* and from Foucault's *The Archaeology of Knowledge*. The results of the analysis, however, are much more in line with Barthes than with Foucault, and are not very far from some hermeneutic approaches.

If we strip down Foucault's archaeology and Barthes' semiotic analysis of their philosophical and political amplitude, and reduce them to bare techniques, it is easy to derive a compact schematic summary. In *S/Z* Barthes traces five dimensions of analysis referred to as *codes*. A text (Balzac's *Sarrasine*) is cut into many short chunks called *lexias*. Each lexia is analysed according to the five codes. The immediate result may appear quite trivial, but a second glance reveals what the codes themselves miss: their interaction. This transformation of the text opens a pathway to an original hermeneutic approach and to interesting reflections on the processes of semiosis that operate through texts.

Barthes' five codes are:

- Proairetic code: actions and behaviours reported in the text;

- Hermeneutic code: questions and enigmas posed by the text, their development and resolutions;

- Semantic code: connoted signifieds evoked to construct characters, atmospheres and other narrative elements;

- Symbolic code: structural relations (opposition, equivalence, etc.) between elements in the text;

- Cultural code: citations of knowledge referred to an ambient culture or to institutions within that culture (academic disciplines, popular wisdom, etc.)

That the articulation of these five codes is arbitrary and underdetermined is explicitly acknowledged by Barthes. For example, the sequential synthesis of individual behaviours into macro actions (such as Barthes' grouping together of the elements 'planning an attack', 'setting an ambush' and 'stabbing' into the sequence of 'murder') is characterised as a **result of an artifice of reading** (Barthes 1974, 19), which depends on the choices of the reader. Rejecting the impulse of structuralist tradition, Barthes chooses to leave the codes as little determined as possible. The Hermeneutic and Proairetic codes are grouped in sequences, but no attempt is made to extract or prescribe the grammar of such sequences; enigmas and action sequences can be broken down and patched together in many different ways. The semantic, cultural and symbolic units are usually left as such; no attempt is made to constrain their various appearances within the text under a certain logic. The result may, at first sight, appear to be a grocery list of arbitrarily articulated elements of text.

The choice of codes, however, is carefully designed in order to reflect both the internal consistency and the openness of the text. The cultural and semantic codes bind the text tightly to its 'outside'. The proairetic, symbolic and hermeneutic codes supposedly emerge from inside the text, but are admittedly manipulated by the reader. The five dimensions of analysis are five axes of interaction between a reader-in-the-world and the letter of the text. These codes reflect Derrida's postulate that **the text has no outside**, in the sense that there is no clear and distinct articulation of

what is definitely irrelevant when reading a given text. Any element of the (supposed) outside may turn out to contribute to the process of semiosis.

The main point of Barthes' analytic technique is to demonstrate how the five semi-arbitrary dimensions of analysis combine together to create effects of meaning and communication with the reader. Barthes thus displaces the focus of analysis from articulating the grammar of each code to observing their relative orchestration. Analysis is no longer the breaking down of a text into small elements and presuming to discover the formal constraints set upon these elements, but rather consists in observing how different elements combine with each other into an effect of meaning. Meaning is no longer identified as a-priori structural constraints, but as a non-prescribed intercourse between arbitrarily articulated units.

Foucault's technique, as laid out in *The Archaeology of Knowledge*,[1] consists in analysing discursive complexes along several formations. The first formation is that of objects. Objects are characterised by the institutions that form them, the places where they are to be observed, and the means by which they are measured and described. But it is not the mapping of objects according to these dimensions that is the focus of interest, as objects of a given discourse need not have common loci or means of observation. It is the constrained relation between various object-characteristics, which is supposed to reflect the unity of a given discourse, and to constrain its object formation.

The other formations that characterise a given discourse include modalities of enunciation (characterised by the questions 'who speaks?', 'from within which institute?' and 'through what means of expression?'), the conceptual formation (the arrangement of enunciations, co-relevance of enunciations, and the acceptable means of transcription), and the strategic formation (the branching of the discourse into several theoretic stances, the interactions between these stances, and the interaction between the discourse and surrounding practices). The archaeologist's task is to find the correlations between the various characteristics of objects, enunciative po-

[1] I focus here on the set of tools introduced in the *Archaeology*'s second chapter, which are far from exhausting the book. The book in fact eventually turns against this simplified structural outlook. For my purposes at this point, however, this limited articulation of questions is extremely useful.

sitions, concepts and strategies that individuate and articulate discourses.

Foucault's technique, like Barthes', breaks the boundary between a textual 'inside' and a practical 'outside'. But, at least at first sight, it appears that this way of analysis is highly structural, as it searches for the limitations and regularities inherent in the various characteristics of a discourse (without assuming, though, that there should be universal structural features common to all discourses). The fact that tips over the supposedly structural attitude is the archaeological choice not to seek **a recollection of the original or a memory of the truth** when delving into a discourse, but rather to actually ***make*** **differences: to constitute them as objects, to analyse them, and to define their concept** (Foucault 1972, 205). Archaeology does not presume to be a purely descriptive tool. Archaeology is explicitly presented in this quotation as a form of intervention. The articulation of objects, enunciative positions, concepts and strategies along their various characteristics is a re-articulation, an action on discourse, anything but an excavation.

Making use of this technical arsenal, but not committing myself to exhausting it or being faithful to it, I analyse Gödel's proof along the dimensions of enunciative position and along the network of transcriptions related to the use of terms such as **mean**, **express** and **signify** in the text. I conclude with a brief analysis of the strategic choices taken in Gödel's text.

1.1 Who speaks?

Alice: **Now, Kitty, let's consider who it was that dreamed it all. This is a serious question, my dear, and you should not go on licking your paw like that — as if Dinah hadn't washed you this morning! You see, Kitty, it must have been either me or the Red King. He was part of my dream, of course — but then I was part of his dream, too! Was it the Red King, Kitty?**

L. Carroll, *Through the Looking-Glass and what Alice Found There*, ch. XII.

A more remote part of the platform. ...
Ghost: ... **but lend thy serious hearing to what I shall unfold**
Hamlet: Speak, I am bound to hear

W. Shakespeare, *Hamlet*, Act I, Scene V.

Who speaks these texts? Which dreamed it? Who has authority over a text? In the quotations above, invariably the king. But not the king as such. It is the dead king, or the chess-piece for the king, who speaks and dreams. Responsibility lies with that displaced king.

But I must protest. The dead king speak? The chess-piece for the king? We're not going to get away with it so easily. When assigning speech to the king, to his symbol, to the dead, we refuse to assume responsibility; authority over speech must be better placed than that. Hence a demand: 'Hamlet, Alice, assume responsibility! it is you who speak! You dreamt it!' But then I must protest again. Alice speak? Victorian girls do not speak. Children should be seen and not heard. Hamlet speak? Hamlet is mad, and the mad are never given the ***platform*** to speak.

Therefore Shakespeare speaks. Carroll speaks. But casting a name as the source of speech does not quite resolve the assignment of authority over text. Shakespeare who? The Earl of Oxford? The Stratford Bard? Or perhaps it is that other person named Shakespeare, the one who actually wrote the stuff, not to be confused with that person named Shakespeare wearing the earring in the portrait? And Carroll, is it Lewis Carroll, writer? Charles Lutwidge Dodgson, mathematician? It might even have been the photographer, Alice Liddell's special friend. Who speaks? Which dreamed it?

It is none other, then, than the signifier who speaks, **a signifier** which **is what represents the subject to another signifier** (Lacan 2006, 694), because **Man is a sign** and **thus my language is the sum total of myself** (Peirce 1931–1958, Vol. 5, §314). But no; we're not going to get away with it so easily. No authority remains with a subject, if it is the king, the character, the name or the signifier who speak. Unless we acknowledge that **We** speak, no ethics can stop us from using foul language.

Who speaks Gödel's proof? Is it Gödel? Is it the sign? Is it Mathematics (**I, truth, speak** (Lacan 2006, 340))? Frege claimed that **the thought for example, which we expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no bearer** (Frege 1967, 29). Young Husserl concurred. **I see that, wherever is talk of the proposition or truth that π is a transcendental number, there is nothing I have less in mind than an individual experience, or a feature of an individual experience of any person** (Husserl 1970a, 329–330). But if that were so, who would be there to command **but lend your serious hearing**? and who would there be to assume responsibility, that is, to respond: **speak, I am bound to hear**? Let us read.

1.1.1 We and I

In Gödel's texts it is a **we** who speaks. In order to study the enunciative position as articulated in Gödel's text, we shall analyse the grammar of this **we**. But before we do, I feel bound to acknowledge that we needn't read too much into this **we**. **We** is simply part of the code. Its use is as imposed upon Gödel as is using **I** when I speak to a friend. So we needn't read too much into **we**, no more, say, than Benveniste reads into **I**.

I, Benveniste explains, is an interface between an individual and language. **Language is so organised that it permits each speaker to *appropriate to himself* an entire language by designating himself as *I***. It is this precise interface, according to Benveniste, which turns the individual into a subject, and without which subjectivity could not be formed. ***I*** achieves that as it **refers to the act of individual discourse in which it is pronounced** (Benveniste 1971, 226). In fact, ***I*** **is "the individual who utters the present instance of discourse containing the linguistic instance *I*."** (Benveniste 1971, 218).

This unified functional view of appropriative **I** would be very convincing, if it weren't for the multitude of different **I**'s appearing in Benveniste's text. Despite the statement of the first quotation above, the ***I***s in the previous paragraph do not function to appropriate language to individuals, but are signifiers referring to this appropriative interface (as witnessed by

the italics and the constructions ***I* refers** and ***I* is**, rather than ‘I refer’ and ‘I am’).

But this is not all. The non actual, non appropriative ***I*** breaks down into smaller constituents. The last quotation above, under pains of circularity, must generate two different grammatical positions (**referent** and **referee**, according to Benveniste). It begins with ***I*** ... **the individual**, and ends with **the linguistic instance *I***. Surely, even if **Man is a sign**, or as Benveniste puts it, **“subjectivity”** ... **is only the emergence in the being of a fundamental property of language** (Benveniste 1971, 224), it is not the individual who is a linguistic instance (at least not according to any point of view that insists on articulating the individual and the subject as distinct).

We so far have an appropriative **I**, which a speaker designates himself as in order **to *appropriate to himself* an entire language**, the reported, non-actualised ***I***, who is the speaking individual who uses this interface, and the **linguistic instance *I***. If only things were so simple! consider that **the indicators *I* and *you* cannot exist as potentialities, they exist only insofar as they are actualised in the instance of discourse** (Benveniste 1971, 224). Here ***I*** is not that which a speaker designates himself as, but that which a speaker designates himself with (the speaker Emile does not designate himself as an indicator when he designates himself as ***I***). Nor is it the uttered linguistic instance, because an indicator is far more functional than a mere linguistic instance. But this ***I***, that is, the indicator, is obviously not the ***I*** who is **“the individual who utters** ... **”**, because indicators are not individual people. We therefore have a fourth, indicator ***I***. But then we’re still not done. The ***I***, which appears in **the indicators *I* and *you* cannot exist as potentialities** can’t be an indicator either, because **indicators** have, in that very sentence, been denied the possibility of existing un**actualised in the instance of discourse**, whereas the indicative function of the last ***I*** is still strictly a potentiality, the very potentiality, which would indicate, when and if it were eventually actualised in discourse. It is, so to speak, the signifier of the indicator.

It seems that whenever we try to articulate **I**, another structural dimension is forced upon it. Whatever instance of ***I*** we are given, science appears to objectify its function, and then signify that object. But there

is yet another, deeper rooted reason for the plurality of **I**'s, which goes beyond the machine of infinite regress. For if ***I*** referred to a particular constant individual, **a permanent contradiction would be admitted into language, and anarchy into its use**. This is especially apparent when considering **I**'s quotability, that is the fact that **If I perceive two successive instances of discourse containing *I*, uttered in the same voice, nothing guarantees to me that one of them is not a reported discourse, a quotation in which *I* could be imputed to another**. On the other hand and at the same time **It is by identifying himself as a unique person pronouncing *I* that each speaker sets himself up in turn as the "subject."** (Benveniste 1971, 218, 220, 226).

I am led to conjecture an extension to Benveniste's claim, which the following paragraphs will attempt to validate: *it is not only intersubjectivity, but substantial components of the very experience of making mathematical sense too, which arise, at least in part, from confounding all these **I**'s and all these functional positions*. In other words, I **shall be that I shall be** (Exodus 3, 14).

1.1.2 What we do

How does the enunciative position in Gödel's text, **we**, fare compared to Benveniste's **I**'s? Gödel's 1931 text contains only three **I**'s, and the 1934 text only one. These **I**'s are used in an acknowledgment, a reference to previous work, a reference to a lecture, and to comment on the relation between Gödel's work and Hilbert's programme. But the person narrating, articulating, and conducting the proof is **we**. And since analysing the grammatic structure of the personal pronoun in the previous section has led us to a futile proliferation of structural positions, let us concentrate on what **we** do. Taking an inventory of Gödel's texts we find that **we** abbreviate, accomplish, add, adjoin, allow, apply, assign, associate, assume, attach, carry out, come, compare, consider, construct, deduce, define, denote, depend, describe, derive, eliminate, employ, establish, exclude, express, find, find convenient, generalise, give, have, include, insert, intend, let, list, make, make use, map, mean, note, observe, obtain, order, proceed, prove, put, replace, require, restrict, say, see, shift, show, sketch, substitute, take into

account, turn to considerations, understand, use, wish, and write.

To make sense of what **we** do, let us follow Foucault, who suggests that a mathematical text articulates various subject positions. **Take the example of a mathematical treatise. In the sentence in the preface in which one explains why this treatise was written** ... **the position of the enunciative subject can be occupied only by the author** ... **only one possible subject** ... **On the other hand, if in the main body of the treatise, one meets a proposition like 'Two quantities equal to a third quantity are equal to each other', the subject of the statement is the absolutely neutral position, indifferent to time, space, and circumstances, identical in any linguistic system, and in any code of writing or symbolisation, that any individual may occupy when affirming such a proposition. Moreover, sentences like 'We have already shown that...' necessarily involve statements of precise contextual conditions that were not implied by the preceding formulation: the position is then fixed within a domain constituted by a finite group of statements** ... **The subject of such a statement** ... **will not be described as an individual who has really carried out certain operations, who lives in an unbroken, never forgotten time** (Foucault 1972, 94).

We may try to make systematic sense of these positions by relating them to the list of verbs above. Different verb types may articulate different functional positions taken by **we**. Fortunately, such typological analysis has already been conducted in the general context of mathematical texts by Brian Rotman in Rotman (1993) and in the first chapter of Rotman (2000). His account, which claims to derive from Peirce, is as follows.

Three characters take part in a mathematical text.

1. The first is called the *Person*, who speaks in the **meta-Code**. He speaks about mathematics, but not in mathematics. He considers the idea or story behind the proof. He has access to natural language, the personal pronoun **I**, and indexicals such as **here** and **now**. His function is to be persuaded by the proof and to understand it.

2. The second character is the *Subject*. The Subject is the entity that defines, derives, considers and proves. His pronouncements are, ac-

cording to Rotman, voiced in a collective imperative: **let us consider, define, demonstrate...** (Rotman 1993, 71). The Subject is abstracted of any temporal, spatial and cultural considerations. **The subject's psychology, in other words, is transcultural and disembodied** (Rotman 2000, 15). The Subject makes mathematical statements, which are, in fact, predictions on the outcomes of sign manipulations. Thus, $x + y = y + x$ is the prediction that if he substitute any number-signs for x and for y, and manipulate these signs according to the orders coded in the '+' sign, the results of the two sides of the equality will be the same.

3. The last character is the *Agent*. The Agent is a reduced image of the Subject, who performs the sign manipulations, about which the Subject makes predictions. He is the one adding, counting, substituting in a sort of thought-experiment or a dream that the subject is experiencing. The **Agent, unlike the Subject, has no ability to imagine and can only respond to signs in their truncated, skeletonised form as signifiers devoid of intentioned meaning. In other words, the Agent is considered as an automaton, a wholly mechanical and formal proxy for the Subject** (Rotman 1993, 76). The Agent's actions are expressed in an exclusive imperative mood (add!, count!, integrate!).[2]

The **Person constructs a narrative, the leading principle of an argument, in the meta-Code; this argument or proof takes the form of a thought experiment in the Code; in following the proof, the Subject imagines his Agent to perform actions and observes the results; and in the light of the narrative, the Person is persuaded that the assertion being proved — which is a prediction about the Subject's sign activities — is to be believed** (Rotman 2000, 35). **The hierarchy of semiotic agencies here, from imagined Agent to imagining Subject to indexically conscious Person structuring a thought experiment is isomorphic to that on which any dream rests: the Agent maps onto the figure dreamed about, the Subject**

[2] An exclusive imperative is an imperative that does not include the person giving the order. 'Eat!' is an exclusive imperative. 'Let's eat!' is an inclusive one.

the dreamer dreaming the dream, and the Person the dreamer awake, consciously interpreting and recognising the dream ... the dream-code ... is restricted in various ways, not least by the lack of the ability to recognise the dream *as* dream, which makes it impossible *for the dreamer* to articulate the dreamer's kinship to the imago he or she dreams into being. Likewise, the restrictive nature of the Code in which the Subject operates, in particular its lack of indexicality, prevents the mathematical Subject from articulating the status of *its* created fiction (Rotman 1993, 78–79).

To explain his division (which is akin to — but does not derive from and is not identical with — Foucault's), Rotman appeals to the imagery of the dream. But his account fails to acknowledge a very unsettling, albeit not too uncommon, occurrence: *lucid dreaming*. Lucid dreaming is the experience, where a dreamer is aware that she is dreaming, and where control over the dream is negotiated between various subjective parts. We must not neglect the possibility that the dream-code may be lucid, and so, occasionally, we should consider waking from the lucid dream. Such experience, I recall, can be extremely unpleasant. Prominent physicist Richard Feynman, however, provides a somewhat more optimistic account for this unpleasant experience. **During the time of making observations in my dreams, the process of waking up was a rather fearful one. As you're beginning to wake up there's a moment when you feel rigid and tied down, or underneath many layers of cotton batting. It's hard to explain, but there's a moment when you get the feeling you can't get out; you're not sure you can wake up. So I would have to tell myself — after I was awake — that that's ridiculous. There's no disease I know of where a person falls asleep naturally and can't wake up. You can *always* wake up. And after talking to myself many times like that, I became less and less afraid, and in fact I found the process of waking up rather thrilling — something like a roller coaster: After a while you're not so scared, and you begin to enjoy it a little bit** (Feynman 1985, 50). Let us, then, muster the courage to elucidate Rotman's **dream**, and talk ourselves awake.

Rotman's divisions are based on both semantic and morphological markers. Morphologically, the Person is the one using the pronoun **I** as well

as indexicals and cultural markers. The Subject is characterised by collective imperatives, and the Agent by exclusive imperatives. Unfortunately, such markers are absent from Gödel's text. Rotman claims that **Even a cursory examination of an arbitrary chosen item of mathematical communication will reveal two fundamental features of mathematical discourse: its organisation as an exhortatory, command-giving formalism and its complete lack of any indexical terms** (Rotman 1993, 71). I do not dispute that if one tries hard enough, one could find such mathematical texts (some concise accounts of geometric constructions might be a good place to look). But our **arbitrary** texts, Gödel's 1931 and 1934 texts (and, as far as I can tell, most mathematical texts), do not conform to these characterisations.

First, there are very few imperatives in these texts. Perhaps the imperative mood was considered ill-suited for civilised written communication in the Vienna and Princeton circles. Perhaps it never occurred to writers in these circles to dominate a mathematical text with the imperative mood. Instead of imperatives, we have the frequent use of the indicative mood in both active and passive voices, often attributed to the character **we**. In addition, the texts contain 9 **here**s and 33 **now**s, some of which would be difficult to marginalise to the **meta-Code**, as they are used to order and demarcate the formal deductive process. These indexicals form part of a network of locative and temporal adverbial structures that operates in the text.

This does not mean that Rotman's analysis has broken down. The lack of morphological markers does not mean we cannot establish a structural division of the enunciative position. We may regroup verbs into subsets, and identify the Subject with occurrences of **we** bundled up with some verbs, the Agent with occurrences of **we** bundled up with others, and the Person as related to yet another set of verbs and adverbials. But here we should take into account one of Barthes' observations concerning the threading of sequences of verbs. **Actions**, he explains, **can fall into various sequences which should be indicated merely by listing them, since the** sequence of actions **is never more than the result of an artifice of reading: whoever reads the text amasses certain data under some generic titles for actions ... and this title embodies the se-**

quence; the sequence exists when and because it can be given a name, it unfolds as this process of naming takes place, as a title is sought or confirmed; its basis is therefore more empirical than rational, and it is useless to attempt to force it into a statutory order; its only logic is that of the "already-done" or "already-read" (Barthes 1974, 19). A formal grouping of verbs risks being an arbitrary empirical compulsion rather than a well grounded structural conclusion.

Bearing this cautionary statement in mind, let's try to allocate some verbs appearing in the texts to the Person, Subject and Agent, and see whether we can distinguish which verb belongs to which character, or whether this distinction is an interpretive framework imposed on the mathematical text. Let's consider three examples. I hope that you will excuse my failure to introduce the details required for a technical understanding of the quotations below. But, as will become evident, the distribution of actions between characters does not depend on such understanding.

EXAMPLE I: **we can, for example, find a formula $F(v)$ of *PM*[3] with one free variable v (of the type of a number sequence) such that $F(v)$, interpreted according to the meaning of the terms of *PM*, says: v is a provable formula**. Footnote: **It would be very easy (although somewhat cumbersome) to actually write down this formula** (1931, 147).

This text appears in the introduction, in a paragraph that opens with the statement: **Before going into details, we shall first sketch the main idea of the proof**, and must therefore be attributed to the Person. But it is obviously not the Person who **find**s formulas. The Person is the one who might try to **write down** the formula, since he is the only one who has enough embodiment to experience how **cumbersome** it is. But **find**ing the formula, which in the context of this example is a tedious formal procedure, is a task which the Subject should narrate and the Agent should perform. Now, which is the character who does the **interpret**ing? Is it the Person who has an overview of all the different layers and significations of the argument, or is it the Subject, for whom interpreting would stand for

[3] *PM* is shorthand for *Principia Mathematica*, Russell and Whitehead's logico-mathematical formal system.

establishing a correspondence between the provability of a meta-statement and the provability of a formula, a correspondence that the Agent can verify by symbolically manipulating the number sequence substituted for v? And once the Person, Subject and Agent have thus collaborated, how come it is the formula, rather than any one of them, which, like Balaam's ass, is suddenly conferred with the power to **say**?[4]

Example II: **we can define relations to be classes of ordered pairs, and ordered pairs to be classes of classes; for example, the ordered pair** a, b **can be defined to be** $((a), (a, b))$ (1931, 153).

The notions **relation**, **ordered pair** and **class** are all already determined from the point of view of the mathematical Person. The authority to represent one by another is therefore contingent on the Person's consent, and it is therefore he who **can define**. But the one to *actually* make the definition, and perhaps prove the formal adequacy of the definition, is the Subject. Finally, it is the Agent, who, whenever hearing the Subject speak of **the ordered pair** a, b must replace such sign sequence by $((a), (a, b))$, and only then carry out his further manipulative tasks. In the Person's voice, the above statement reads: I agree to define. In the Subject's voice it reads: I define (and, perhaps, verify that the definition is adequate). In the Agent's voice it reads: I make the symbolic manipulations required by the definition.

Example III: **We have noted that** $x\mathcal{B}y$ **is a recursive relation; and we can also prove that** $\sigma(x, y)$ **is recursive, where** $\sigma(x, y)$ **is the number of the formula which results when we replace all free occurrences of** w **by** z_y **in the formula whose number is** x (1934, 359–360).

This statement seems to have some verbs relating to the Subject (**note** and **prove**), and some to the Agent (**replace**). However, it also contains temporal and locative elements. The noting is articulated as having occurred in the past. This is a psychological and relative past. If for instance, I skip

[4]Some logicians object that formulas don't say, and shouldn't be described as saying. This normative stance is irrelevant here. I am analysing how a specific mathematical text works. According to this text, Formulas do say.

the text's discussion of recursive relations, and move directly to the main argument (as in fact I first did), then my reading Subject and Gödel's writing Subject get out of synch, and manifest a temporal relativity, whereas the mathematical Subject is supposed to be free of such relativity. Gödel's Subject's present-perfect **have** is my Subject's future-simple **will**. This desynchronisation is in fact explicitly supported by the text, which states that the discussion of recursive functions is **a parenthetic consideration that for the present has nothing to do with the formal system *P*** (1931, 157). The discrepancy between Gödel's Person and mine has seeped through to create a discrepancy between Gödel's Subject and mine. The embodied Person is allowed to carry the Subject along with it.

On the other hand, the locative preposition **in** is, as far as the text is concerned, an absolute disembodied location, which is invariant to any Subject and Agent (I do not claim that it is an absolute invariant, but only that the text articulates it as such). That this locative be a disembodied invariant is demonstrated by the text's claim that such manipulations can be mechanised. However, even Rotman allows a restrictedly embodied manifestation of locality into the mathematical code, as provided by the *zero* point of reference. **The point to be made here about "0" is that in practice its two senses — Coded number and metaCoded origin — are inextricable from each other: in the course of manipulating the number sign, the meta-sense is always present shadowing it, being part of another layer of meaning which adjoins and penetrates the formal layer available in the code** (Rotman 1993, 75).

The three examples above *are* analysed according to Rotman's articulation, however problematic and obscure the analysis turned out. There is, however, nothing *in* the text to impose such analysis. Gödel's verbs and pronoun **we** do not seem to distinguish between the Person, Subject and Agent, but rather to bundle them seamlessly together. We saw that a single verb can order into action all three characters, and that their realms of authority are not clearly distinct. On top of this obscurity, a writer who were to take as a point of departure Foucault's division quoted above, would likely end up with a somewhat different character distribution.

I find no explicit textual evidence that such triple articulation permeates the text. And yet, we cannot ignore the fact that we *can* indeed

rewrite the text so as to conform to Rotman's articulation. The fact of this capacity must not be dismissed, because it is still a fact *of* the text. It is still a fact of the text that such a reading can be sustained, and as such this fact may very well be relevant to the question of how the text works. This fact can contribute to our understanding of how the text interacts with possible readers, even though it is no more a fact *of* the text than the effect of an *outside* intervention. The fact that a structure, which is not *in* the text, can be imposed *upon* the text, that such imposition does effect some changes, but does not entirely break the text apart, and that such imposition can appear, at least at first sight, quite convincing — these facts testify to a critical feature of mathematical texts: openness.[5] This is the same sort of openness, or rather tolerance, which allows the extraction from (or imposing on) mathematical texts of platonist, logicist, formalist and intuitionist positions — even though these positions needn't be discoverable from analysing mathematical texts according to terms they explicitly state.

What we impose on the text from the outside is not without precedent inside. The roles suggested by Foucault and Rotman, as well as platonist, logicist, formalist and intuitionist metaphysics/epistemologies are not completely foreign to the text. They are not arbitrary chance constructs that have nothing to do with the mathematical text. They can all be supported to an extent by evidence excavated from within the text. Indeed, If this had not been the case, no one would have been able to successfully impose these constructs on mathematical texts, or at least not in ways that appear convincing to the extent that they do. The mathematical text communicates with various structures, which are not completely foreign to it, but which are not properly endorsed by it either. This is precisely how a mathematical text can signify: by communicating with its outside, by incorporating inside a residue of its outside, by denying its outside the clear and distinct metaphysical status of outside.

This diagnosis of openness lies well within the range spanned from Barthes' and Foucault's notions of authorship in Barthes (1977) and Fou-

[5] An entirely different question is *for what purpose* we should want to impose upon a text Rotman's (or any other) articulation. Rotman uses his imposed structure to argue for a radically finitist view, which I respect, and, with some modifications, endorse. Rotman's rearticulation has its merits. But it is not 'discovered' within the text.

cault (1984), through Eco's conception of *The Open Work* (Eco 1989), to Derrida's maxim that **there is no outside-text** (Derrida 1976, 158). The novelty is in demonstrating that mathematical texts are not outside this range. Without such openness, without the text's incorporation of externally imposed enunciative positions, the text would simply not communicate with 'foreign' readers and hence soon fail to signify. But imposing upon **we** the three (Person, Subject, Agent) who are one is still awkward and contrived, and it is the interpretive gap between the text and the reconstruction that I seek to bring up and explore. Facing **we** with the question **What is thy name?** one should not confine to the asylum the possibility of **saying, My name is Legion: for we are many** (Mark 5, 9).

1.1.3 When, where and how we do it

Here is what we learnt from our critical engagement with Rotman's three characters (or more generally, from Barthes' comments on grouping verbs together): that the text tolerates, at a price, divisions that are imposed on it, and allows trace evidence of such divisions to be excavated from within it. This openness to an outside (which undermines a clear inside/outside division, because it blurs the line between imposing something from the outside and discovering it inside) is a crucial constituent of the text's semiotic capacity, but leaves us without a concrete analysis of the text's enunciative position. We shall therefore try to determine this position via the adverbials and modalities interacting with this position.

Temporality makes its mark on the text in various forms. Temporality appears in formal manipulations, which may take place one **after** another, as in: **we must therefore (1) eliminate the abbreviations and (2) add the omitted parentheses** (1931, 156). Temporality also appears along the course of the argument, as in: **We have noted that** $x\mathcal{B}y$ **is a recursive relation** (1934, 359), and in **As will be shown later, however, the converse does not hold** (1931, 173). Finally, temporality intervenes in reviewing and interpreting the text: **such a proposition involves no faulty circularity, for initially it [only] asserts that a certain well-defined formula ... is unprovable. Only subsequently (and so to speak by chance) does it turn out that this formula is**

precisely the one by which the proposition itself was expressed (1931, 152).

But to appreciate the nature of this temporality, an important clue is provided by the transmutation of 'outside world' temporality into 'mathematical' temporality. This happens in the following passage: **Suppose that on 4 May 1934, A makes the single statement, "Every statement which A makes on 4 May 1934 is false." This statement clearly cannot be true. Also it cannot be false, since the only way for it to be false is for A to have made a true statement in the time specified and in that time he made only the single statement** (1934, 362).

This form of the liar paradox is then translated into mathematical form. Recall that Gödel constructed an explicit enumeration of formulas in his formal system. Then a formula is constructed claiming that formula number so-and-so has a certain property F (interpreted as being false). The construction is so designed that the number of that formula is the very number so-and-so mentioned inside the formula. The formula therefore says of itself that it has property F (namely, is false). But where in the transition from **4 May 1934** to the self-referential statement did temporality disappear?

Temporality has been converted into enumeration. The date **4 May 1934** served only to designate A's claim. This designation device is transformed into Gödel's enumeration. The form of temporality acknowledged here by the text is so reductive, it is not even properly serial. The numbers conferred upon formulas have nothing to do with the order in which they appear. Time is but a pool of events and designations, which excludes co-occurrence, but which does not display any equivalent of duration (some possible exceptions will be treated below).

But the most interesting aspect is not the articulation of time into a set of mere mutual exclusions, but the ambiguous relation between the enunciative position and this temporality. On the one hand, the enunciative position is within time, as the above quotations explicitly mark. However, the same position has the capacity to review the entire temporal pool. What **will be shown** and what **we have noted** is readily available for the current moment of the discussion. Whatever happened **initially** and **sub-**

sequently in the quotation standing four paragraphs above is bundled together in a contemporary moment where the proposition is self-referential, but **involves no faulty circularity**. Even hypothetical moments of an unspecified time can all be bundled up and reviewed together: for any relevant system of axioms κ, **there always are propositions** ... **that are undecidable** ... **as soon as** some specific proposition **is not κ-PROVABLE** (1931, 195).

The enunciative position, which is within time, and at the same time can observe time and even form it by the operation of enumerating its minimal elements — this complex position contributes to the complex process of semiosis. First, temporality as a set of mutual exclusions registers the distinct elements (formal propositions) studied by the mathematical text. Second, temporality as a sequence of past and future occurrences helps order and narrate the text for the reader. Finally, by taking a stance that can inspect all these different times, Gödel counters the grave accusation that his argument is groundless, and shows that his **proposition involves no faulty circularity**. In so far as temporality is reduced to the articulation of mutual exclusive elements, there is a sort of circularity (self reference) — the referent formula and referee formula are one and the same. But in so far as temporality has a before and an after, in so far as **initially it [only] asserts** ... and **Only subsequently (and so to speak by chance) does it turn out** ... — in that sense we obtain a non-circular turn of events.

The extraction of meaning from the text depends here on instilling a difference across the self-referring instance. The fact that the order of arguing and reading is not absolute, but revisable by any reader, does not weaken the argument that depends on its distinct temporal sequencing. The fact that one has to adopt an omni-temporal point of view in order to complete the story does not weaken it either. But the enunciative position must be able to endorse *all* these conceptions of temporality in order to put Gödel's argument together. The argument's validity depends on a multiplicity within the enunciative position. If the enunciative position were severed into several positions, each endorsing only one conception of time, the argument would simply not stick together.

Some researchers insist on discarding the narrative, and reading the proof 'purely formally', without retaining notions of reference and expres-

sion operative in the proof. This reading is of course tenable, but leaves the reader in a senseless position. In this reading all that Gödel's theorem provides us with is a couple of highly complex arithmetic propositions concerning some very big numbers. Unless these numbers are allowed to refer to formulas, there is no incompleteness theorem. But we may go even further: if we refuse to let signs refer, all we have is a bunch of symbols that obey some combinatorial restrictions — not even an arithmetical statement. But we shall leave this strand of the discussion to the third chapter.

Going from the notion of time to that of space, we find that space in the text is usually reduced to relative locations in sequences. This relative, disembodied notion of space is further abstracted by formulations such as: **a formula will be a finite sequence of natural numbers** which is accompanied by the footnote: **That is, a number-theoretic function defined on an initial segment of the natural numbers. (Numbers, of course, cannot be arranged in a spatial order)** (1931, 147). The spatial concept of **sequence** is here reduced to a function providing an ordinal 'location name' for every element in the de-spatialised sequence. The underlying spatial configuration is dismissed and marked as impossible due to the non spatial abstraction of numbers.

Spatial notions in the text are rarely related to the enunciative position. Two of the few exception are: **This makes no difficulty in principle. However, in order not to run into formulas of entirely unmanageable lengths** ... **the construction of the undecidable proposition would have to be slightly modified** (1931, 149) and **The proof that** so-and-so holds **is too long to give here** (1934, 359). Here the physical limitations of space, and likely also time (or perhaps spatialised-time), suddenly intervene.

The discrepancy between this last notion of space-time and the ones observed immediately before are better accounted for by an analysis of modality in the text. The text appears to distinguish difficulty **in principle** from **actual** difficulties. Consider, for instance, that **we always understand by "formula of** the formal system ***PM*" a formula written without abbreviations (that is, without the use of definitions). It is well known that [in *PM*] definitions serve only to abbreviate notations and therefore are dispensable in principle** (1931, 147). It

is obvious that abbreviative definitions cannot be dispensed with, if mathematics is something to be practiced by physically and culturally constrained humans of a kind we tend to meet. The above statement, therefore, serves, rather than to make a claim, to articulate the field where the argument is to apply. The argument is to apply in the realm of **in principle**, which we shall later relate to other conceptual layers of the proof. One should bear in mind that in practice mathematicians hardly ever write proofs in as precisely articulated formal systems as ***PM*** at all.

Similar considerations apply to the already quoted statement, which appears on the same page: **It would be very easy (although somewhat cumbersome) to actually write down this formula**. Easy, perhaps — but not humanly feasible. Today it may be feasible to program a computer to output such a formula, but extremely cruel, and quite likely unfeasible, to expect a human to examine the output. Either way, this option was not available to Gödel. Again, we have a statement that operates in the language game of the **in principle** grammar, where things can be done that cannot *actually* be done.

We may appear to have found a textual support for the Agent vs. Subject-Person division. It is the disembodied, unlimitedly industrious Agent, who can do whatever can be done **in principle**. The Person, perhaps even the Subject, are excluded from this capacity. But whereas we do have two distinct modalities in the text, there is no indication that the enunciative position is in fact divided according to these modalities.[6] Consider for instance the statement (also on the same page) **it can be shown that the notions "formula", "proof array", and "provable formula" can be defined in the system *PM*; that is, we can, for example, find a formula $F(v)$ of *PM* ... such that $F(v)$ ... says: v is a provable formula.** The first **can** will in fact be achieved in the text. But the next **can** is precisely the kind that is only achieved **in principle**, and the last **can** falls somewhere in the middle, depending on what aspect of **find**ing is at stake. The enunciator must endorse both modalities if it is to attain

[6] Claiming that any grammatical division (or even that any modal division) necessarily entails a division of the enunciative position leads to grotesque results. The imperative and indicative moods are distinct, but they do not necessarily require distinct enunciative positions to function properly. Indeed, Austin's *How to Do Things with Words* ends up questioning the distinctions between indicatives and performatives.

both human accessibility (actual capacity) and the mathematical ideal (**in principle** capacity). The text does not show any sign that the enunciator relates differently to these seamlessly interwoven layers of capacity — and yet these layers are distinguished by the text itself, when it briefly insists on the **in principle** reservation.

What I have been trying to show is that the texts make sense and lend themselves to the extraction of meanings by, on the one hand, supporting, to a certain extent, divisions of the speaker, while, on the other hand, refusing to be pinned down to such divisions. The enunciative position has been shown to be distributed across various roles — but without actually grounding any stable textual articulation of such roles; it has been shown to endorse various incompatible modalities of temporality, spatiality and capacity — and then bind them together into intelligibility by avoiding clear distinctions followed by explicit syntheses. The text relies on many diverse positions to explain itself, to make the reader understand; but the text cannot commit itself to any single position, or set these positions as entirely apart, as by doing that it would simply fall apart.

The text manages to produce its meaning by distributing differences across the repeated enunciative stance. The semiotic machine embodied by the morpheme **we** is, like Benveniste's **I**, a blurring interface that ties a plurality of readers, writers and their forms of being to texts in ways that make sense. Regardless of popular belief, mathematical texts, such as these, do not make sense by pure formal clarity. They make sense by having it both ways (and more).

1.1.4 Who do we do it with? (a slight detour)

Whoever **we** are, they can't stand alone. **I use *I* only when I am speaking to someone who will be a *you* in my address. It is this condition of dialogue that is constitutive of *person*, for it implies that reciprocally *I* becomes *you* in the address of the one who in his turn designates himself as *I* ... This polarity of persons is the fundamental condition in language, of which the process of communication, in which we share, is only a pragmatic consequence** (Benveniste 1971, 224–225).

A polar position to the texts' enunciative position is established in the very first lines of the introduction to the 1931 text. **The development of mathematics toward greater precision**, Gödel explains, **has led, as is well known, to the formalisation of large tracts of it, so that one can prove any theorem using nothing but a few mechanical rules. The most comprehensive formal systems that have been set up hitherto are the system *Principia mathematica (PM)* on the one hand and the Zermelo-Fraenkel axiom system of set theory on the other** ... **These two systems are so comprehensive that in them all methods of proof are formalised, that is reduced to a few axioms and rules of inference. One might therefore conjecture that these axioms and rules of inference are sufficient to decide *any* mathematical question that can at all be formally expressed in these systems. It will be shown below that this is not the case, that on the contrary there are in the two systems mentioned relatively simple problems in the theory of integers that cannot be decided on the basis of the axioms** (1931, 145).

The footnotes to the second sentence acknowledge Whitehead and Russell, Fraenkel, von Neumann and Hilbert and Bernays as authors of formal systems. These various authors are invoked as if they belong to a single unified front, binding together the fundamentally logicist agenda of Russell, Hilbert's finitism, Zermelo's realist point of view and von-Neumann's pragmatist formalism, subjugating all to an ideological field negatively marked by the adjective **mechanical**. This adjective, along with the verb **reduce**, suggests a non-voluntary, non-subjective and impoverished mathematical practice. Indeed, at the time of writing the paper machines had already had the kind of connotation to be canonically presented in Chaplin's *Modern Times* (1936). At the same time, the use of the term **mechanical** in the context of formalism is far from obvious. A mechanical approach will only be established a few years later by Church and Turing.

In the above quotation a position of 'they' has been set-up and characterised. 'They' are ascribed a mighty achievement. Due to 'their' work, **one can prove any theorem using nothing but a few mechanical rules**. An unsuspecting reader may be lured to believe that a certain saturation has been achieved within **large tracts** of mathematics, whereby

given a theorem, a proof can (mechanically!) be produced. And indeed, those readers who swallow the bait[7] **might therefore conjecture that these axioms and rules of inference are sufficient to decide *any* mathematical question that can at all be formally expressed in these systems** (1931, 145).

Note that the constative assertion is reduced by the hypothetical modal **might**. The text, however, does not yet clarify to the possibly misled reader that the formalist project allows to transcribe and verify by **few mechanical rules** only those proofs that are *already discovered*. Unproven and unrefuted theorems remain unproven and unrefuted regardless of the formalist project. The modal shift from constative to hypothetical serves to muddle the reader, who is now watching the conjecture he was manipulated into forming subjected to the doubt inscribed in a **might**. And indeed, **It will be shown below that this is not the case, that on the contrary there are in the two systems mentioned relatively simple problems in the theory of integers that cannot be decided on the basis of the axioms** (1931, 145). But is it an oversight that an explicit definition of undecidability (there exists a statement such that neither it nor its negation can be proved) is postponed to the bottom of the next page? The specialist may have known exactly which undecidability Gödel was talking about. Other mathematicians were probably still kept in suspense.

The rhetorical structure we have just reviewed is standard. A picture is painted, then questioned, and finally announced invalid. Let's analyse it in Barthes' terminology. The first chunk of quoted text (*lexia*) paints a picture through *Cultural Coding*: a reference to institutional knowledge (the work of the formalist 'school'), and of *Semantic Coding*: the negative signified connotation of mechanical reduction. The second lexia appeals to *Hermeneutic Coding*: the modal shift operated by the **might** sets up a lure to undermine the very picture painted. Finally, we resort to *Symbolic Coding*: a pivotal antagonistic **this is not the case**, and *Proairetic Coding*: the action of **show**ing by an implicit agent, in order to invalidate the former

[7]The translation appears to be more misleading than the original, which speaks of *Beweisen* (proofs) rather than theorems. But even the original claim is stronger than what was considered as commonly accepted at the time, and indeed, Gödel himself casts a doubt on this claim at the end of the paper (1931, 195).

picture. This minor triumph of Barthesian analysis is almost too neat to accept. The emerging message of deliverance is reflected not so much in the bits of information provided by each of the codes, but in the very passage from one to the other: from institutional knowledge and connoted negativity, through a mystery, to a symbolic rapture by an agentless act. This is the structure of deliverance.

The enunciative position in the text takes advantage of this manoeuvre. For the **It will be shown** to emerge both as a legitimate offspring and a parricidal revolutionary with respect to a certain lineage, a paternal position has to be, in the space of a few lines, both established and denounced. This position is established by forcing together Russell, Hilbert, Fraenkel and von Neumann into a straw-coalition, ignoring the fact that formalism was extremely young,[8] highly unstable, and far from resting on a consensus among the contributors named above. This paternal position immediately falls victim to a parricide performed by branding this straw-formalism with the suspect mark of mechanism, and assigning to it the ambiguous and misleading formulation **axioms and rules of inference are sufficient to decide *any* mathematical question**, which formalists would not have dared to claim as achieved.

The positioning accomplished here relies on the ability of a standard rhetorical manoeuvre to impose itself on a discursive field that is still underdetermined and emerging. This move provides the rhetorical support (which is, of course, not the only support) for putting it, later in the text, that **The solution suggested by Whitehead and Russell, that a proposition cannot say something about itself, is too drastic** (1934, 362).

Indeed, the tension between belonging to, and breaking away from 'them', which gives rise to the text's impersonal **it** and collective **we** enunciative positions, is particularly manifest in the manoeuvres around the **proposition that says about itself that it is not provable [in *PM*]**, which, as a footnote we have already quoted explains, **involves no faulty circularity, for initially it [only] asserts that a certain well-defined formula ... is unprovable. Only subsequently (and so to speak by chance) does it turn out that this formula is precisely the one by**

[8]Merely 13 years had passed from Hilbert's 1918 formulation; see Kleene's introduction to Gödel's 1931 paper in Gödel (1986–2003, 126).

which the proposition itself was expressed (1931, 151). The enunciative position endorses here at once the aversion from and attraction to self-reference. No one would deny that the self-referential proposition was constructed with the explicit intention of emulating self-reference inside a formal system. And yet the mere possibility of telling a revisionist history, obviously false *as a history*, relating the accidental genesis of this self-reference, is sufficient to allow contemporary logic both to embrace Gödel's form of self reference,[9] and to be revolutionised by it. The text remains formalist. But into this **mechanic** realm of formalism quietly sneaks an element of **so to speak** ... **chance**.

But all this rhetorical mechanics and revisionist historical articulation of meta-mathematical positions relates to the preamble to the introduction, and, for fear of being accused of lurking in the margins, we must move back into the core of the mathematical text.

1.1.5 Who does it to us?

A formalist 'they' position was set-up in order to simultaneously counter and produce the text's enunciative **we** position. But this 'they' position is far from exhausting the stance polar to the enunciative **we**. In order to complete the articulation of this polarity, let us track down the few occasions where **we** is demoted from the subject position to the object position **us**.

First, we must acknowledge how rare this move is. In both texts there are only nine occurrences of **us**, of which three appear in the inclusive imperative **let us**. Nevertheless, these occurrences are revealing of the presence of the enunciative position in the much more frequent impersonal and passive constructions.[10]

[9] There are, of course, logicians who altogether deny that Gödel's proposition is self-referential. But I don't believe anyone would deny that self-reference was a **leading principle** in the construction of the proposition.

[10] I should have, perhaps, analysed the impersonal and passive constructions in the text. Indeed, Foucault writes that the subjective formation **is situated at the level of 'it is said'** (Foucault 1972, 122). However, since the relation between some of these constructions and the enunciative position is debatable (for instance, in statements of the form 'it follows that...'), I preferred to focus on the textual occasions where the voice emanating from the enunciative position is explicitly marked as such (lest I be accused

We are told that **if a formal decision** ... **of the SENTENTIAL FORMULA** 17Gen$r$ ... **is presented to us, we can actually give** ... **a PROOF of** Neg(17Gen$r$) (1931, 177). Ignoring the meaning of 17Gen$r$, we observe that polar to **we** a position is established, which may present **us** with mathematical offerings. Confronting this position, **we** becomes a reactive (**giv**ing once **present**ed), rather than an initiating instance.

But this polar position is not confined to a **giv**ing role. **We** can **have before us a proposition that says about itself that it is not provable** (1931, 149–151) — the polar position can speak. In fact, more than just speaking, it can modify the capacities of the enunciative position. A certain proof, for example, **allows us to actually derive a contradiction** ... **once a PROOF of** w ... **is given** (1931, 195) (without getting into what it is, such a **PROOF of** w is supposed not to exist).

We is therefore a subject position in both senses of the word: subject of and subject to. However, the **proposition that says**, which we **have before us**, is explicitly produced by **we** in the text. Even the hypothetical object **PROOF of** w above, if provided at all, must necessarily be provided by someone who can present proofs, and therefore share **we**'s enunciative position. **We**, it seems, plays a game of catch with itself, or rather, as is now obvious, with themselves.

We have many voices, many positions from which **we** speak. **We** speak to and of ourselves and our creations, who in turn, as the above quotations demonstrate, do not hesitate to speak back and challenge. In some sense (a sense I wish to impose) **we** are none but Gödel and I, the reader, in the most intimate moment of reading — intimate but not private, because I know many others have done it with him too, and because, to a certain extent, here I am doing it with him in public. Perhaps these so many discursive partners are the cause of this text being infested with so much meaning. Having intercourse with so many codes (encoded and decoded by so many partners), it is bound to say many things. **We**, which is supposed to establish a single common denominator, ends up, it seems to me, forcing the text open to many different entangled codes and complicit partners. So many different entangling codes indeed, that it is no longer clear that one could speak of *any* individual code at all.

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of hearing voices).

I would like to conclude this discussion with the words by which Barthes described the ambiguous enunciative position in Balzac's short story *Sarrasine*. **Who is speaking?** he asks. **Is it a scientific voice** ... **Is it a phenomenalist voice naming what he sees?** ... **Here it is impossible to attribute an origin, a point of view, to the statement. Now, this impossibility is one of the ways in which the plural nature of a text can be appreciated** ... **it may happen that in the classic text, always haunted by the appropriation of speech, the voice gets lost, as though it had leaked out through a hole in the discourse. The best way to conceive the classical plural is then to listen to the text as an iridescent exchange carried on by multiple voices, on different wavelengths and subject from time to time to a sudden *dissolve*, leaving a gap which enables the utterance to shift from one point of view to another, without warning: the writing is set up across this tonal instability (which in the modern text becomes atonality), which makes it a glistening texture of ephemeral origins** (Barthes 1974, 41–42).

I could use Foucault's words to conclude this section. **In the proposed analysis, instead of referring back to *the* synthesis or *the* unifying function of *a* subject, the various enunciative modalities manifest his dispersion. To the various statuses, the various sites, the various positions that he can occupy or be given when making a discourse. To the discontinuity of the planes from which he speaks. And if these planes are linked by a system of relations, this system is not established by the synthetic activity of a consciousness identical with itself, dumb and anterior to all speech, but by the specificity of a discursive practice. I shall abandon any attempt, therefore, to see discourse as a phenomenon of expression — the verbal translation of a previously established synthesis; instead, I shall look for a field of regularity for various positions of subjectivity. Thus conceived, discourse is not the majestic unfolding manifestation of a thinking, knowing, speaking subject, but, on the contrary, a totality, in which the dispersion of the subject and his discontinuity with himself may be determined** (Foucault 1972, 54–55). I would like to endorse this quote, but I am not

sure that the terms **regularity**, **totality**, and **may be determined** are pronounced here with the cautious and critical tone in which I would like to have heard them.[11] I am worried that this quote underplays the capacity of a textual **we** to disperse and integrate, regularise and breach, totalise and re-open, determine and render undecidable. If we do not underplay these capacities, we can perhaps acknowledge not only the rigorous aspect of mathematics, but also its disruptive creative force. And so I shall not be satisfied before I come again (now divulging in a voice drenched with the fluid complexity of so many enunciative distributions compacted into a brief contractive exhale: **we** — no longer opposing a trinity, but embracing its charge, embracing it *too*) to exclaim **My name is Lesion: for we are many**.

1.2 The object: notion

Foucault's methodology considers objects and concepts as distinct elements that require separate analyses. However, forcing discursive objects and concepts apart in mathematical texts is usually less straightforward than maintaining their distinction in other contexts of knowledge (such as psychiatry, linguistics or economy). In mathematics what we speak about, and what we say about it tend to intermingle. We may study numbers through the concept of order, but then go on to study the concept of order as a mathematical object and subject it to quantitative classifications, turning numbers from objects to regulative concepts.[12]

In the texts under our consideration the problem is even more acute. Gödel's texts manifest an explicit and persistent motif of objectifying concepts (or, in his terminology, notions). Therefore, rather than assume that objects and concepts can be distinguished and im/exposing modalities of such articulation on/in the texts, I would rather bundle objects and concepts together. As Foucault suggests, I will analyse statements (which observe objects and operate concepts) from the point of view of transcription, that is as elements **that men produce, manipulate, use, transform,**

[11] They most likely are so pronounced in other parts of *The Archaeology*.

[12] This object/concept tension has been widely discussed in the literature. Two of the most prominent relevant references are Frege (1983) and Benacerraf (1983a).

exchange, combine, decompose and recompose, and possibly destroy (Foucault 1972, 105). More precisely, I will analyse the roles of the verbs **mean**, **express**, **interpret**, **represent**, **correspond**, **denote**, **signify**, **say** and **understand** in recasting relations between objects/notions and statements in the texts.

1.2.1 Abbreviate notations

The most trivial use of the verbs above is in introducing definitions that **abbreviate notations**, and are therefore, according to the text, **dispensable in principle** (1931, 147). Typical examples include statements such as ***Bew*** x **means:** x **is a provable formula** and **we denote the** n**th** class sign in the sequence of all class signs **by** $R(n)$ (1931, 149). The verbs **say** and **understand** are also used to the same effect, as does the verb **represent**, which may occupy such a role, as indicated by the parallelism **German letters will be used in abbreviation for finite sequences of** natural numbers **i.e.,** $\mathcal{X}$ **for** $x_1, \ldots, x_n$**;** $\mathcal{Y}$ **for** $y_1, \ldots, y_m$**. Greek letters** $\phi, \psi, \chi \ldots$ **will represent functions** (1934, 347).

The trivialising view of such **abbreviate notations** is most markedly expressed by the parallelism **If** ϕ **denotes an unknown function, and** $\psi_1, \ldots, \psi_k$ **are known functions** ... (1934, 368). To denote and to be are collapsed here to one and the same status.[13] This trivialising view of abbreviate notations as completely lucid was criticised as circular in Quine's classic *Two Dogmas of Empiricism* (Quine 1961, chapter 2). Quine does exclude from his claim abbreviations in strictly formal languages; in Gödel's text, however, we can see the dangers of treating abbreviate notations too naïvely, even when one is very close to the strictly formal context.

One of the abbreviations that Gödel introduces is the following: **We abbreviate certain formal expressions as follows:** z_0 **for** 0**,** z_1 **for** $N(0)$**,** z_2 **for** $N(N(0))$**, etc.** (1934, 358) $N(x)$ here is the formal representation of the successor of x, i.e. of $x+1$. Instead of writing a numeral the way it should be written in the formal system, say $N(N(0))$ for the number 2, one writes the shorthand z_2. There seems to be no danger here. z_2 does

[13] Unless one reads here an ontological statement, that the unknown can only be denoted, whereas what is known actually is.

not belong to the formal system, but whenever we see it, we know which text of the formal system must replace it.

But things become more delicate when we encounter z_p, where p is the extremely big and uncomputed number of a certain complicated formula. The supposedly naïve abbreviation enables Gödel not only to write a text that would otherwise be too long to write in a publishable article (not to mention an impossible obstacle for a referee), but also does away with the need to conduct a complicated computation. What goes on here is not simply saving time and effort, but also rendering possible what would otherwise be practically impossible, a qualitative leap that enables the writer and reader to avoid completely, and not only to abbreviate, a certain computation.

But the qualitative leap from the numeral in the formal system to the supposed abbreviation does not end here. Consider the statement $N(z_n) = z_{n+1}$ on the same page, where n is a variable. This 'abbreviated' statement is impossible to expand into a text in the formal system. Indeed, while z_n contains the variable n, the formal system has no means of expressing a sequence of embedded N's of a variable length. This 'abbreviated' statement therefore contracts infinitely many statements of the formal system. It does not simply abbreviate *an* expression, but essentially enriches the expressive abilities of the writer. Of course, one could reconstruct z_n as a place holder for a variable in the formal system (say, w), rather than for a numeral that depends on a variable (even though this is not how Gödel presents it). In that case one would expand $N(z_n) = z_{n+1}$ into the formal statement $N(w) = N(w)$, as there is no 'more primitive' way to express the right hand side. The original statement $N(z_n) = z_{n+1}$, which is not an explicit tautology, turns into an explicit tautology. The **abbreviate notation** turns out to be more manipulative and less naïve than would seem at first sight. To say that abbreviate notations are **dispensable in principle** is to use a very extreme form of that **in principle** grammar, which we reviewed in the previous section.

This observation suggests how the use of a supposedly simple abbreviation almost imperceptibly crosses the line between languages, and how it forms a first step in confounding variables and constants (a concern to be further pursued in the eighth section of the next chapter). Indeed, where

we think of z_n as containing a variable n, this n is a variable of arithmetic rather than a variable of Gödel's formal system, and therefore z_n is not an abbreviation of a text in the formal system, but the application to a variable of a function from the realm of non-negative integers into the formal system. If, alternatively, we insist on maintaining that this n is a constant, then we must confront such phrases as **each natural number** n (1934, 358), which make the border between constant and variable very fluid indeed.

I am not claiming that there is a formal error in the proof, because one can easily circumvent the problem via a proper logical reformulation. Since the problem can be easily circumvented, from a logical point of view my observation is nothing but nit-picking of marginal interest. I nevertheless insist upon this issue here, because from the point of view of a critical reading this issue is suggestive of the distance between the actual text and a supposedly underlying correct logical reformulation. What I reject is a reading that ignores the surface text and restricts itself to the supposedly underlying formal text, which might be practically unwritable or at least, were it to stand alone, inaccessible for mere mortals of our limited capacities. It is the gap between actual text and the shadow of the supposedly precise formal text (especially as this gap interacts with the observations concerning the more complex transcription devices to be surveyed below), that bestows upon the bigger picture the productive blurriness I wish to bring to the foreground as enabling the text's intelligibility. Note that this productive blurriness is effectuated in part by using the same verbs (**mean**, **represent**, **denote**, **say** and **understand**) to express the above (relatively) simple form of transcription and the other more complicated transcription forms to be surveyed below.

1.2.2 From primitive signs to numbers

A major innovation and an essential pivot point of Gödel's manoeuvre consists in relating *metamathematical* notions to *arithmetic* ones. Such a move requires, of course, a preliminary articulation of both fields. **Notions which relate to the system considered purely formally**, the text posits, **may be called *metamathematical*** (1934, 355). Metamathemati-

cal notions include **for example, "variable", "formula",**[14] **"sentential formula"**,[15] **"axiom" ,"provable formula", and so on** (1931, 157).

The licence to transform the metamathematical into arithmetic comes from the fact that **for metamathematical considerations it does not matter what objects are chosen as primitive signs, and we shall assign natural numbers to this use. Consequently, a formula will be a finite sequence of natural numbers** (1931, 147). There is no metamathematical objection, the text proclaims, to replace the primitive symbols $\forall$, x and $\rightarrow$ by the numbers 1, 2 and 3. Metamathematics is thus articulated as the discursive domain that considers systems purely formally, but is ignorant of the choice of primitive signs; it considers the outward appearance of formal expressions, but not the specific content of this outward appearance.

This vague situation, however, turns out to be not sufficiently felicitous for the text, and is therefore amended by two footnotes on the same page. First, rather than **assign natural numbers** to the role of **primitive signs**, **we map the primitive signs one-to-one onto some natural numbers**. And since **Numbers, of course, cannot be arranged in a spatial order** (they are, after all, abstract notions), a formula turns out to be not a **finite sequence of natural numbers**, but a more abstract construct: a function that assigns numbers-as-place-names to numbers-as-mapped-onto-primitive-signs-taking-these-places.

These amendments reflect the stance that numbers are not just any objects **chosen as primitive signs**. Rather, they impose their abstraction (not being able to take a concrete physical place) upon the formal system. I bother with this nit-picking, because this is our first evidence as to the manner in which the texts cross over from the signifier as a concrete object (a **primitive sign** that can be written on paper) to the signifier as a structural abstraction or platonic object (a number). The proof will carry with it this double[16] setting.

[14] A **formula** here is a sequence of signs that obeys the syntactical requirements of the given formal system. One should think here of a 'proposition', 'statement' or 'expression' rather than of a formula for computing or constructing something.

[15] A formula without free variables.

[16] At least double! We've already seen how such settings tend to proliferate when discussing Benveniste's **I** and Austin's **shoot** above.

However, we shall have to be more specific in order to reflect the full extent of the relation between metamathematics and arithmetic that is established in the texts. The texts describe an explicit dictionary, which translates each sign of a formal system into an integer (not unlike children who make up codes by replacing each letter of the alphabet by some other sign). As a result, a formula in the formal system, which is a sequence of signs, can be translated into a sequence of integers. Then some arithmetic procedure is applied to transform each such integer sequence into a single integer, in a way that guarantees that no two formal expressions are transformed into the same integer. This **representation**, **correspondence**, **understanding** or **expression** of **meaning**[17] is a mechanico-computational process of translation. Claims about formal objects ('the sequence of signs XXX is an axiom', 'the sequence of signs XXX is a proof of the sequence of signs YYY') become, via this transliteration, claims about numbers ('the number x represents a sequence of signs which is an axiom', 'the number x represents a sequence of signs which is a proof of the sequence of signs represented by the number y').

The arithmetic **relations between (or classes of) natural numbers that in this manner are associated with the metamathematical notions defined so far, for example, "variable", "formula", "sentential formula", "axiom", "provable formula", and so on, will be denoted by the same words in** SMALL CAPITALS (1931, 157); that is, rather than say 'the number x represents a sequence of signs which is an axiom', we say 'the number x is an AXIOM'; rather than say that 'the number x represents a sequence of signs which is a proof of the sequence of signs represented by the number y' we say that 'x is a PROOF of y'. This typographic device articulates two distinct domains: the lower case metamathematics, and the small-capitals arithmetic.

However, rather than transliterating between two stable and independent domains — metamathematics and arithmetic — this textual device

[17] I make a point of noting the verbs used in this context to indicate that the text does not attempt to segregate the various mechanisms of transcription documented in the various subsections of this section by distinguishing specific verbs for each form of transcription. This is part of the rhetorical technique of simultaneously distinguishing and conflating discursive modes in order to produce meaning, that is to assert repetition across differences.

appears, at certain times, to uproot and haul the entire edifice of metamathematics into arithmetic; not just a transliteration, but rather a full transubstantiation. The introduction specifies that **metamathematical notions (propositions) thus become notions (propositions) about natural numbers or sequences of them** (1931, 147). This statement, perhaps not entirely clear, is further detailed in a footnote, which explains that **the procedure described above yields an isomorphic image of the system *PM*[18] in the domain of arithmetic, and all metamathematical arguments can just as well be carried out in this isomorphic image. This is what we do below when we sketch the proof; that is, by "formula", "proposition", "variable", and so on, *we must always understand the corresponding objects of the isomorphic image***. Here we no longer have metamathematics transliterated into arithmetic, but, rather, metamathematics discarded or abandoned in favour of its image as a sub-domain of arithmetic.

Many references in the texts to the metamathematical are ambiguous, and conform with either interpretation (transliteration and transubstantiation). It does seem, however, that the 1931 text tends towards keeping metamathematics and arithmetic distinct, and prefers to operate in the arithmetic domain, whereas the 1934 text leans towards blurring the distinction between metamathematics and its arithmetic image, and operates sometimes in one domain, sometimes in the other, and sometimes treats both domains at once.

This confounding of domains seems to relate closely to a fact of the texts, which, although stated explicitly and obviously, took me quite a few readings to properly absorb. The texts do not quite attempt to mathematically prove that some formal systems contain an undecidable formula (namely, a proposition such that neither it nor its negation are provable). What the texts do prove mathematically is that there are numbers, x and y, which have the property of being FORMULAS, such that one of which has the arithmetic property of being the NEGATION of the other, and such that both x and y have the arithmetic property of being NOT PROVABLE. There is no attempt to control *mathematically* the transliteration/transubstantiation of formal systems into arithmetic, and to show that the arithmetic statements

[18] ***PM*** is Principia Mathematica, the main formal system Gödel uses in his argument.

indeed imply the existence of undecidable formulas in formal systems.

Before we explain this seemingly broken link, let us make this gap more manifest. I claimed above that the assertion 'the number x represents a sequence of signs which is an axiom' is an **arithmetic** assertion. This would be true only if we could express this claim using strictly arithmetic tools (logical connectives and operators, the equality predicate, and the addition and multiplication functions). The text does explicitly present such transliteration. A slow and gradual process allows to translate all the metamathematical notions, which are relevant to the argument, into arithmetic notions. However, when, for instance, the text defines the operation $x * y$ (which, given the number x of a sequence of signs XXX, and the number y of the sequence of signs YYY, returns the number corresponding to the concatenation of the sequences: $XXXYYY$), an arithmetic definition is given, which is then followed by the comment $x * y$ **corresponds to the operation of "concatenating" two finite number sequences** (1931, 165). What is missing is an attempt to validate that the arithmetic formula indeed corresponds to the metamathematical operation. Such attempt is not only missing in this and in other specific occurrences. Nowhere do the texts attempt to mathematically validate the claimed correspondence.

It is crucial to explain here that I make no claim that the correspondence assumed between metamathematics and arithmetic should be marked as invalid. I do, nevertheless, insist on pointing out that the text *does not manifest a need to validate the correspondence.* The text contracts two a-priori distinct domains: the material domain of text (formal relations between signs) and the ideal domain of numbers (arithmetic relations between numbers). But the link that is supposed to validate the contraction is not given.

It is easy to excuse this 'broken' link. In Gödel's view the actual formal system and its metamathematical study are not mathematical objects. The term **meta** reflects, in this case, an outside. Obviously, there can be no *mathematical* validation of a link between something mathematical and something non-mathematical. The link is to be established by Gödel and the readers, who are to acknowledge a valid but non-mathematical link. It is only after we have managed to carry over metamathematics into arithmetic that we can start to analyse things mathematically.

This formative attitude allows for the manifest ambiguous relation between metamathematics and arithmetic in the text. The transfiguration of the former into the latter is meant both to articulate and set metamathematics and arithmetic apart, and at the same time to suppress the former and incorporate it into the latter.

The braid of the two understandings, says Barthes, **creates an equivocation. And in fact the equivocation results from two voices, received on an equal basis: there is an interference of two lines of destination. Put another way, the *double understanding*** (*double entendre*), **the basis for a play on words, cannot be analysed in simple terms of signification (two signifieds for one signifier); for that there must be the distinction of two recipients; and if** ... **both recipients are not given in the story, if the play on words seems to be addressed to one person only (for example, the reader), this person must be imagined as being divided into two subjects, two cultures, two languages, two zones of listening (whence the traditional affinity between puns and "folly" or madness: the Fool, dressed in motley, a divided costume, was once the purveyor of the *double understanding*). In relation to an ideally pure message (as in mathematics), the division of reception constitutes a "noise," it makes communication obscure, fallacious, hazardous: uncertain. Yet this noise, this uncertainty are emitted by the discourse with a view toward a communication: they are given to the reader so that he may feed on them: what the reader reads is a countercommunication; and if we grant that the *double understanding* far exceeds the limited case of the play on words or the equivocation and permeates in various forms and densities, all classic writing (by very reason of its polysemic vocation), we see that literatures are in fact arts of "noise"; what the reader consumes is this defect in communication, this deficient message; what the whole structuration erects for him and offers him as the most precious nourishment is a *countercommunication*; the reader is an accomplice, not of this or that character, but of the discourse itself insofar as it plays on the division of reception, the impurity of communication** (Barthes 1974, 145).

I hope to have demonstrated, and to continue to demonstrate, how even **in mathematics** there is no, *pace* Barthes, **ideally pure message (as in mathematics)**. I hope to show how **noise** and **double understanding** operate at the heart of a mathematical text to produce **countercommunication**. How **folly** and the plurality of speakers/readers is operative in the process of semiosis without imposing a simplistic duality on the text. This would not mean that mathematics fails, or that it would be any less useful. It would, however, mean that the hypothesis that mathematics succeeds to the extent that it does *because of its supposed* ***purity*** — this hypothesis will find itself undermined.

Here we have extracted two models that act simultaneously: in the one the grammar of metamathematics (I mean here, of course, that part of metamathematics that is imported into the arithmetical language), explicitly articulated as a-priori non-mathematical, is made parallel to a sub-grammar of arithmetical grammar; in the other model metamathematics is already an arithmetic sub-grammar operating inside arithmetic. I claim here that the process of semiosis in the text involves a simultaneous attempt to keep these two models apart, and rely upon both.

Indeed, the first model suggests two separate manoeuvres: the first of which would be a non-mathematical endorsement of a relation between a material and an ideal domain, and the second would be a mathematical analysis, which no longer has anything to do with the non-mathematical domain (the metamathematics of formal systems). If we were to endorse this first model alone, then Gödel's argument would be a combination of a non-mathematical argument (correspondence between metamathematics and arithmetic) and a mathematical argument (the proof of the arithmetic analogue of undecidability). We could then make no mathematical claim concerning the undecidability of formal systems as such.

If, on the other hand, we were to endorse the second model alone, then we could no longer rigorously segregate mathematics from metamathematics. This is a problem, as the formal articulation of language and metalanguage rules over the entire argument as it is laid out in Gödel's text. Without this articulation, for example, the distinction of the semantic and syntactic argument collapses, as this distinction is all about whether a certain universal quantifier is to be attributed to the language or to the meta-

language (see the section *Gödel's argument in brief* in the introduction). More importantly, if we reduce meta-language to its image inside mathematical language, it is much more difficult to argue against the **faulty circularity** of the proof in the way that Gödel does, as it would then involve an immediate, rather than an indirect form of self-reference (see page 85 below).

Each model threatens to upset an important aspect of the text. The first threatens mathematical jurisdiction over its meta, whereas the second threatens the independence of mathematics from its meta. The confounding of the two models into one seamless ambiguity is a crucial component for enabling Gödel's end-product: a *mathematical* analysis of a *metamathematical* object.[19] True, this ambiguous combo still upsets the boundary between mathematics and its outside; but at least it allows mathematics to say something about formal systems. The argument works, not despite this upsetting of boundaries, but due the incorporation of these upset boundaries. Ambiguity here is not destructive, but rather conductive and productive of meaning.[20] And I would like to show that the textual formation operated in Gödel's texts is in fact even more complex and productive than is suggested by this initial ambiguity.

1.2.3 From numbers back to signs through proofs

The manoeuvre we have just surveyed, complex as it is, is only half the story. Now that we are in the abstract realm of arithmetic, we have to re-encode everything into a formal system, in order to guarantee that our proof can be carried out in full rigour according to the prevailing standards

[19]This situation has some bearing on the debate concerning the distinction between synthetic and analytic statements (e.g. Putnam 1975, chapter 2), and may therefore shed some light on the history of the relationship between Gödel and the Vienna circle.

[20]In this context the following observation by historian of mathematics Jacob Klein suggests itself (his **concepts of the first** and **second class** are roughly equivalent to what we might call 'concepts' and 'meta-concepts' or 'concepts about concepts' respectively; for Klein numbers are concepts of the first class and variables are concepts of the second): **in algebra we use concepts of the second class as though they were concepts of the first class ... what we call a symbol is nothing else but a concept of the second class interpreted as a concept of the first class** (Klein 1985, 62–63).

of 1930s formalists and intuitionists. We are already laden with a double understanding (arithmetic and metamathematical) of the terms **"formula", "proposition", "variable"**, when the plot thickens, and we are told that these doubly determined notions **can (at least in part) be expressed by the symbols of the system *PM* itself. In particular, it can be shown that the notions "formula", "proof array", and "provable formula" can be defined in the system *PM*** (1931, 147).

Gödel does not take for granted that his metamathematical/arithmetic notions can indeed be expressed by the limited resources of the formal system *PM*. Unlike the previous transliteration/transubstantiation, this transformation from the language of arithmetic into the formal system *PM* is not left unvalidated. In fact, the most technical and tedious part of the argument is devoted to showing that such translation is possible for the relevant metamathematical/arithmetic claims. The extent to which this translation is not taken for granted is reflected in the comment (apparently made to reconcile the supporters of Hilbert's programme) that **it is conceivable that there exist finitary proofs that cannot be expressed in the formalism of *P***[21] (1931, 195). But despite (or perhaps because of) the care invested into managing this translation, it too does not escape multiplicity.

The first criterion that regulates the transformation of an arithmetic statement into a statement in the language *PM*, the one most carefully articulated, relies on provability. Let $n, m, k \ldots$ be integers, and $z_n, z_m, z_k \ldots$ stand for the signs that represent them in the formal language *PM*. Given an arithmetic function Φ and a function G in the language *PM*, **we shall say that the formal functional expression $G(u_1, u_2, \ldots)$ represents $\Phi(x_l, x_2, \ldots)$ ... if $G(z_m, z_n, \ldots) = z_k$ is provable formally whenever $\Phi(m, n, \ldots) = k$ holds** (1934, 358). Two mathematical procedures are to be related, if this **representation** (sometimes also referred to as **expression** (1934, 359,361)) is to be valid: the arithmetic *computation* of Φ and the *formal proof* of statements about G. The text in fact promises a procedure, which, for some arithmetic functions Φ (the so called **recursive functions**, which we needn't define here), constructs representations G in

[21]For our purposes the reader may ignore the differences between the various formal systems that Gödel refers to, in particular *P* and *PM*.

PM, and transforms the computations of Φ into formal proofs concerning G.

This formulation is careful not to have anything to do with **meaning**, whatever **meaning** may mean at this point. This point is explicitly stated in the text. To introduce the theorem, which guarantees the **represent**ability of recursive arithmetic functions by formal expressions, the text explains that the **fact that can be formulated vaguely by saying that every recursive relation is definable in the system *P* (if the usual meaning is given to the formulas of this system) is expressed in precise language, without reference to any interpretation of the formulas of *P*, by the following theorem** (1931, 171). The formal expression G will be equivalent to the arithmetic function Φ not because of what they mean, but because the computation of the latter can be transformed into a proof concerning the former. And therefore, one may alter somewhat the text of the representing G (even at the cost of slight shifts of meaning), as long as the computation–proof link is maintained. This slight abuse of notation is made explicit by the statement claiming that **If the value of $\Phi(x_1, x_2, \ldots)$ is independent of some variable x_p, then $G(u_1, u_2, \ldots)$ need not contain the corresponding variable u_p** (1934, 358).

It is somewhat strange, then, that when the text is supposed to prove this representability of arithmetic functions by formal expressions, it appeals to the fact that some formula **intuitively has the desired significance** (1934, 359), and therefore **means** whatever it should mean for the argument to work. Rather than prove what is promised, the 1934 text regrets that the proof is **too long to give here**. The 1931 text too regrets that it will **give only an outline of the proof of this theorem because the proof does not present any difficulty in principle**[22] **and is rather long** (1931, 171), and in a footnote on the next page explains that when **this proof is carried out in detail** a certain element in it **of course, is not defined indirectly with the help of its meaning but in terms of its purely formal structure**.

I must hasten to say that my mathematical training leads me to cast

[22]For a discussion of the **in principle**, consult the third subsection of the previous section of this chapter.

no doubt on the conjecture that a complete formal proof can, in principle, be given. But I am not willing to discard as insignificant the fact that such a proof is not in fact given, and that the syntactic operations of computation and formal proof are put in abeyance inside the cadre of semantic explanation. This is exactly the kind of gap between actual and hypothesised text, which allows to confer one sort of understanding on another phenomenon, forcing repetition across differences, conducing intelligibility. That an exact formal proof can, in principle, be written I do not doubt. But that such a proof can stand alone without the accompaniment of a semantic explanation I find hard to believe. However, since the problem of a 'purely formal' text will be considered in the final chapter of this book, I leave it aside for now.

Before we turn to analyse the articulation of semantic meaning, which the text fails or neglects to adequately set apart from the above formalist notion of **representation** or **expression**, we should point out how the ambiguity considered in the previous subsection interacts with the ambiguity considered here.

Representation is supposed to convert an arithmetic expression and computation into a formal expression and proof. This conversion is meant to link the arithmetic and formal points of view by going from arithmetic to a new formal coding, rather than going back from arithmetic to the original metamathematical claim. However, we have noted above that one should not be able to prove mathematically a statement about a metamathematical notion such as **expression** or **proof**, since this may result in 'crossing the line' between mathematics and its meta. And when we discuss the provability of a formal coding of an arithmetic relation expressing a metamathematical claim, the same problem reappears.

The 1931 text takes this objection more seriously than the 1934 text. The 1934 text simply states that given a valid arithmetic relation, the provability of the relation's representation by a formal expression follows (1934, 358–359). Theorem V of the 1931 text (page 171), however, states that given a valid arithmetic relation, the arithmetic property PROVABLE holds for the number encoding the formal expression. In the 1931 text, we make metamathematical claims, translate them into arithmetic ones, code these arithmetic claims formally, and then encode the provability of these

formal claims arithmetically again. Theorem V then states that the earlier arithmetic relations imply the latter ones. The refusal to deal with a metamathematical notion unless it is turned arithmetical, for a moment, takes the upper hand. But by 1934 the need for a second arithmetic re-encoding of objects in and claims about formal systems is ignored, except, perhaps (this is not quite clear), in the footnote on page 360, added in the 1964 publication, which says that the formal expression that was constructed **is *not* the undecidable sentence, but only *denotes* it**.

This 30 years late clarification serves only to deepen the ambiguity built into both texts. If the text is to be finite, the process of translation must somewhere come to an end. One has to work, eventually, either in arithmetic or in a formal system. The 1931 text ends up working in arithmetic. But such an abstraction may be frowned upon by formalists, who require mathematics to be done in a formal system. The 1934 text ends up working at the level of formal texts, but thirty years later reopens the question. The double-entendre regulating the metamathematics-arithmetic interface interacts here with the carefully articulated formal representation of arithmetic in formal systems to produce a potentially infinite chain of translations.

The decision to stop at a certain level — formal or arithmetic — is arbitrary. One more translation is always implicit. One more translation may always be lurking inside the text only to be made explicit 30 years later. To point out the gravity of the situation, one should understand that an arithmetic-to-formal-to-arithmetic translation *does not* end up with the original arithmetic statement. The original arithmetic statement or number is very different from the number produced by the double translation. Gödel's translation processes are not inverses of each other. Further translations will continue to render resulting objects more and more complex.

The issue is that nothing compels us to stop at the arithmetic level. Should we, perhaps, take the arithmetic statement, express it in a formal language, and prove it in this language? But why stop there? Why not use Gödel's numbering to encode the latter proof and theorem as numbers, and validate the arithmetic statement that a PROOF relation holds between these two numbers? And why not state and prove this arithmetic claim formally? And why not encode everything arithmetically again? Obviously,

Explanations must **come to an end** (Wittgenstein 1953, §1). But this fact doesn't prescribe any specific endpoint.

Trying to simultaneously maintain formal and platonic points of view puts the argument into a perpetual spin. One more translation is always implicitly there. On the other hand, trying to keep to a single point of view (either arithmetic-platonist or metamathematical-formalist) would leave us with no explicit validation either for the link between the ideal arithmetic and concrete formal application (in the platonist case), or between the formal text and what it is supposed to signify or represent (in the formalist case).[23]

Gödel chooses both at once. For Gödel the link between ideal arithmetic and concrete formal text is valid because the ideal arithmetic domain and concrete formal domain are *formally* isomorphic (**an isomorphic image of the system *PM* in the domain of arithmetic**). The link between the purely formal text and its interpretation (a proof of undecidability) is valid because Gödel can resort to a platonically-inspired common ideal source for the purpose of linking them together (as he states, **the meaning which we attach to the symbols is a leading principle in the setting up of the system**).

Surely, the platonic-formalist combo is not the only possible solution. Wittgenstein, for instance, argues in his *Lectures on the Foundations of Mathematics* for a conventional and practical link between the various ways of computing, proving and interpreting, rather than an 'ideal' or 'structural' one.[24] I am also well aware that devoted formalists and devoted platonists can defend their stances at least every bit as successfully as I can undermine them, so I do not argue for the necessity of Gödel's platonist-formalist combo. In fact I do not make any metaphysical argument at all. I merely track down how a double-entendre is operative in the generation of meaning in Gödel's text, on the text's terms.

I am presumptuous enough to conjecture that effects of meaning depend on an at least double *entendre*: the superimposition of more than one point of view and the constitution as the same of the different. I do not aim to prove this conjecture, because I cannot imagine a system that is

[23]One of the most celebrated discussion of such dilemmas is Benacerraf (1983b).

[24]See footnote on page 104.

rigorous enough to justify the term 'prove', and general enough to contain the entire scope of the conjecture. In fact, I cannot even imagine a system that can formulate the above conjecture as a rigorous conjecture. But as a regulative idea, I find that this claim has a lot to do with how mathematics works.

1.2.4 What is the meaning of it all?

Back to the text. We found an operative ambiguity in the relation of the metamathematical discussion of a formal system and its arithmetic image (independent strata translatable into each other, or a non-mathematical domain suppressible in favour of an arithmetic one). Maintaining this ambiguity is crucial for a mathematical treatment of a metamathematical claim. We also found that in justifying the translation from arithmetic into a formal system, semantic arguments (based on the meaning of symbols) took the place of formal considerations, even though formal considerations should and could **in principle** be applied. We also pointed out an indecision as to where to terminate the process of translation between arithmetic and formal languages that is operative in the argument. We then went on to assert the generative power of this indecision, and the way it crossbreeds formalism and platonism into a meaningful mathematical analysis of formal systems.

Next we would like to show how the first ambiguity (allowing to maintain *and* infringe upon the boundary between mathematics and metamathematics) cross-breeds with the role of semantics in an argument that should, **in principle**, be syntactic. To do that we shall try to understand the grammar of the term **meaning** in this complex formation.

One vein in the text constructs **meaning** as formal and arbitrary. It is **easy to state with complete precision *which* sequences of primitive signs are meaningful formulas and which are not** (1931, 147). If it is indeed easy, it is because **meaningful**ness is defined here as an arbitrary feature of formal systems. **A *formal mathematical system* is a system of symbols together with rules for employing them. The individual symbols are called *undefined terms*. *Formulas* are finite sequences of the undefined terms. There shall be defined a class**

of formulas called *meaningful formulas* (1934, 346). Just so opens the 1934 text. But surely, some restriction must be made on this arbitrary set of ***meaningful formulas***. Indeed, on the same page we **require that ... the definitions of meaningful formulas ... be constructive; that is ... there shall be a finite procedure for determining whether a given formula A is a meaningful formula**. Here **constructive** is a formal requirement. **Meaning**, here, is simply the effect of (or a title for) this formal requirement.

But meaning is not always this formal effect. **While a formal system consists only of symbols and mechanical rules relating to them,** explains the text, **the meaning which we attach to the symbols is a leading principle in the setting up of the system** (1934, 349). Meaning is not *in* the system, but it has actually been *around* all along. That is how **undefined terms** can actually stay **undefined** and at the same time have meaning. And that is why the statement that the **undefined terms, in addition to variables, shall be ...** $\neg, \vee, \&, \rightarrow, \equiv, \Pi$ **(**$\Pi x(F(x))$ **means "**$F(x)$ **is true for all natural numbers** x**", and may be regarded as the logical product of F(x) over all x)** (1934, 350) — that is why this statement can appear safely without provoking protests against the apparent oxymoron between **undefined** and the following definition preceded by the term **means**.

But the relation between a formal system and its meaning is best expressed by the claim that at a certain point in the argument, **the meaning of the symbols is immaterial, and it is desirable that it be forgotten** (1934, 355). **Meaning**, it appears, is a sort of clip-on. It was there from the **setting up**, but it can be detached and re-attached at will (or **desire**).

I shall spare the reader the quotations, which demonstrate that **meaning** can be attached to formulas in the formal system as well as to arithmetic expressions, that it can be preserved despite some formal modifications (such as binding a variable that does not have a free appearance in a given formula), and that one can **express** it, **interpret according** to it, and even replace it by the terms **say**ing, **significance**, **correspond**ence, **express**ion and **represent**ation.

But this clip-on meaning, which clips-on to various terms and systems,

and which is replaceable by many of the other terms that operated in our discussion above, is perhaps somewhat more constrained and imposing than it first appears to be. In the 1964 postscript to the 1934 version, the text suggests that the term **finite procedure**, which appeared at the beginning of the 1934 text, should mean **mechanical procedure**. It goes on to claim that this **meaning, however, is required by the concept of formal system, whose essence it is that reasoning is completely replaced by mechanical operations on formulas** (1934, 370). Meaning here is constrained by some prevailing **concept** and **essence**. This preeminent **meaning** can also constrain formal notions like provability. Indeed, **We have seen that in a formal system we can construct statements about the formal system, of which some can be proved and some cannot, according to what they say**[25] **about the system** (1934, 362).

One could raise technical objections to using the above quotations as evidence for the claims that meaning is constrained by something prior to it (**essence**), and that formal properties (such as provability) are constrained by meaning, rather than strictly by **finite procedure**s of verification. But one cannot escape the fact that meaning is *sometimes* added arbitrarily, and but at other occasions has always-already-been-there.

One may hypothesise that we're dealing with two different notions, which only accidentally, so to speak, share the same term; that in fact the former should be entitled *artificial* or *formal* meaning, and the latter *proper* meaning. But a simple example can refute this hypothesis. Consider the arithmetic operation $x*y$, which takes the number x representing a formula XXX and the number y representing a formula YYY, and returns the number that represents the concatenation of the formulas: $XXXYYY$. As we have already observed, there is no attempt to justify that this arithmetic operation, defined by the equation

$$\begin{aligned} x * y \quad \equiv \quad & \varepsilon z\{z \leq [Pr(l(x)+l(y))]^{x+y} \& \\ & (n)[n \leq l(x) \rightarrow nGlz = nGlx] \& \\ & (n)[O < n \leq l(y) \rightarrow (n+l(x))Glz = nGly]\}, \end{aligned}$$

does indeed **correspond** to concatenation. The justification is folded *within the proper meaning* of the formal terms. But this generation of meaning

[25]It is easy to demonstrate that in this section of Gödel's text the verbs **say** and **mean** are used as synonyms.

(meaningitis?) presupposes the representation of a formula XXX by a number x, which is an *artificial* ad-hoc constructed representation, and which is only *formal*ly constrained. The **correspond**ence depends on the seamless cooperation between the *artificial* and *proper* meanings. The above formula means whatever it is intended to mean only if these two forms of meaning build upon each other to produce an integral meaning. Even if we insist that *artificial* and *proper* meanings are different terms, the meaning of $x * y$ is a hybrid of the two forms of meaning. The full **meaning** of the text is created only if we allow the two hypothesised formations of meaning to advance past their differences.

But even before these two hypothesised formations of meaning meld into an integral formation, each of these meanings already contains the other. When the text constructs the arbitrary transcription of formulas into numbers, it already builds on the *proper* meanings of the words and signs that are used to introduce the *artificial* meaning. And in order for the **undefined terms** of the formal system to express their *proper* meaning, the one which was **a leading principle** in their articulation, an arbitrarily directed correspondence had to be imposed on specific signs (the $N(\cdot)$ of the 1934 was an arbitrary choice for the successor function $(\cdot)+1$, in the sense that it could have been replaced by many other conventional signs, such as the f of the 1931 text), and an arbitrary form of number representation (unary[26] rather than, say, decimal) had to be enforced. As the two hypothesised meaning formations depend on each other, and combine to produce an integral meaning, I no longer see any use in hypothesising two separate meaning formations. I therefore endorse Gödel's unified terminology for a complex notion of **meaning**.

The discussion above refers to the tension between traditional and ad-hoc relations between symbols and meanings. But even the relation between meaning as expressed content and meaning as the formal constraint of constructivity (the existence of a finite mechanical procedure for checking whether syntactic rules are obeyed) cannot be properly set apart. A formal system is built with an aim to express content, that is with meaning

[26]A unary representation of numbers is a representation based on an alphabet consisting of only one repeatable sign. If this sign is, say, 1, then the number 5 would be represented as 11111.

as a **leading principle**. But it is also constructed under the demand of constructive syntax, which is also a leading principle. These two principles obviously constrain each other. The tension between 'natural' and 'formal' languages is located precisely in the interaction between the two requirements. Expressible meaning is reduced and distorted precisely in favour of a constructive syntax. Meaning in a formal system is already a hybrid product of two leading principles.

On top of the ambiguities internal to the transcription mechanisms surveyed in this section (abbreviate notations, representation of metamathematics by arithmetic, representation of arithmetic in a formal system, and meaning), we must note that these transcription mechanisms are not morphologically articulated by different terms. The terms **representation**, **expression**, **meaning**, etc. each carry a reference to more than one form of transcription. In some uses of these terms it is not perfectly clear which transcription mechanism they refer to. This suggests further interdependence between the different modalities of transcription, complex intercourse and hybrid descendants.

The texts do not signify *despite* these ambiguities. My purpose is to point out that it is *because* of these ambiguities that the texts signify successfully. Semiosis is not only an effect of the segregative articulation of these forms of transcription, but also of their internal multiplicity and instability, and of their cross breeding to a point where it is no longer possible to tell, on certain occasions, which belongs where and which gave rise to which; where it is no longer possible to limit in advance their potential for further cross-production.

Gödel's text is not unaware of the collapsing borders, and makes an attempt to rescue these borders from the onslaught of nomadic modalities of transcription. **We therefore have before us a proposition that says about itself that it is not provable [in *PM*]** (1931, 149–151), states the text. In order to **say** something like that, the process of **say**ing to which Gödel's undecidable formula is subject must take into account the entire edifice of interweaving **meaning** formations and modalities of transcription, and to recall that meaning which, at some other point, **it is desirable that it be forgotten** (1934, 355). In order to rearticulate this apparently muddled interaction, a footnote states: **Contrary to appearances,**

such a proposition involves no faulty circularity, for initially it [only] asserts that a certain well-defined formula (namely, the one obtained from the qth formula in the lexicographic order by a certain substitution) is unprovable. Only subsequently (and so to speak by chance) does it turn out that this formula is precisely the one by which the proposition itself was expressed (1931, 151). According to this statement, first one transcription operated, and then, by chance, when we applied another form of transcription — lo and behold our surprise — the proposition turns out to refer to itself. Everything here is properly articulated into distinct serial steps.

This reconstruction is too little too late. The first interpretation alone (**a certain well-defined formula** ... **is unprovable**) already requires to combine practically all transcription devices described in this section. And to claim that the subsequent identification of that formula as self-referential is, **so to speak**, a **chance** event, is a very unconvincing disappearing-reappearing act. The beautifully chimeric combinations of forms of transcription are all components in the genesis of the self reference, which — it would be impossible to deny — was in fact a **leading principle in the setting up** of the formula.

True, one could, perhaps, re-edit the argument such that the effect of self reference will emerge as a surprise to the reader; this potential editing must not be ignored. But neither can the fact be ignored that this is not the case; that the texts opted not to surprise the reader; that no text of Gödel's proof that I know of ever attempted such narrative; and that **chance** has to do with the **leading principle** of the argument only in a **so to speak** kind of way. The text is meaningful because the event reconstructed as **chance** was predestined from the onset. Even if the texts pretend to set their transcriptions apart, in a time outside the time of the texts, in the time of the writer and of the readers, the different transcriptions depend on each other and cross into each other. What we sought to demonstrate above is how these different transcriptions remain entangled even inside the very text which supposedly sets them as independent and distinct.

1.2.5 Conclusion

You can never go back home; a formal system may be nothing but signs and syntactic rules, but when statements about the formal system are transformed into arithmetic statements about numbers and are then re-transcribed into the formal system we started with, it no longer is the formal system we started with. Because now this formal system is doubly-impregnated with **expression**, with **representation** and with **meaning**.

Every sequence of primitive signs is articulated according to the grammar of formalism, according to the grammar of arithmetic, according to its meaning as a **leading principle**, and according to their interdependence and cross-breeding. Formulas no longer simply *are*, they also *say*, and they say in cross-bred codes and tongues, in an unbounded, uncontainable field of voices and potentialities — the cross products of the grammars articulated above with each other and with their readers. And now reading the texts according to this inarticulate mush of codes and transcription mechanisms is no longer a mere contingency; now it is constitutive of the proof. Without blurring the carefully laid boundaries the proof won't be intelligible. The entire argument is an elaborate double — or rather multiple — entendre. It is still valid, but its validity relies on its multiplicity.

An important question remains only partly confronted: is the semiosis of the text possible *because* we have, to begin with, a set of articulated codes, which later become interdependent and cross-productive, or do we a-posteriori reconstruct an articulation of codes derived from an always-already cross-bred border-crossing, pulling the rug from under the hypothesis that we actually have an underlying system of separate codes to begin with? This will be a main concern for the two following chapters.

The following chapters will also confront the claim that the argument does not depend on a double-entendre or self reference at all; that once we formalise the entire argument, and display it in a purely formal language, all issues of (double) meaning disappear. I don't doubt that a formal transcription is possible in principle. I will argue that it *alone* cannot signify in the way that the actual texts do, and that unless the formal text is burdened with some manipulative intervention, unless it takes upon itself the burden of a multiple meaning, it will not be able to signify at all.

To conclude this section, I feel it is best to point to a constituent of

the fragile mechanism that allows to separate-and-breed, or articulate-and-confound, the various formations of transcription employed by the text. This mechanism is exposed by the wonderfully telling indication, which suggests that **the meaning of the symbols is immaterial, and it is desirable that it be forgotten** (1934, 355).

One may indeed desire to forget, especially things immaterial.[27] But the actual act of forgetting is impossible, while one is deliberately desiring to forget. One cannot forget something while willing to forget it (forget the phrase 'pink elephant'!). Forgetting and desiring to forget are mutually exclusive. How does one desire to forget? Does one state 'I wish to forget'? Does one count sheep to force the would-be-forgotten out of one's mind? Or is it that 'desiring to forget' is akin to 'pretending to forget'; one acts as if one has already forgotten, while in fact one actually remembers. This pretended forgetfulness is precisely the sort of interaction between various forms of transcription, which allows them to breed and interact while maintaining a claim — a claim without which the texts would be hard-pressed to function successfully — to stay apart.

The freedom to forget is one of the essential factors in the openness of the text to the reader's semiotic intervention. For a certain kind of reading, a kind of reading that I attempt to demonstrate is possible (I dare not say compulsory) even for a mathematical text, forgetting is not only an opportunity, but is actually a constitutive act. — **With regard to the plural text, forgetting a meaning cannot therefore be seen as a fault. Forgetting in relation to what? What is the *sum* of the text? Meanings can indeed be forgotten, but only if we have chosen to bring to bear upon the text a singular scrutiny. Yet reading does not consist in stopping the chain of systems, in establishing a truth, a legality of the text, and consequently leading its reader into "errors"; it consists in coupling these systems, not according to their finite quantity, but according to their plurality (which is a being not a listing): I pass, I intersect, I articulate, I release, I do not count. Forgetting meaning is not a matter for excuses,**

[27] In German, Gödel's mother tongue, more so even than in English, immaterial (*immateriell*) suggests the opposite of material, rather than the opposite of relevant. Meaning, the text can be read to quietly suggest, is not material.

an unfortunate defect in performance; it is an affirmative value, a way of asserting the irresponsibility of the text, the pluralism of systems (if I closed their list, I would inevitably reconstitute a singular, theological meaning): it is precisely because I forget that I read (Barthes 1974, 10–11, translation modified).

1.3 A strategic point of diffraction

One of the reasons that Gödel's text attracts so much attention is that it is a crucial point of diffraction in the history of logic and mathematics. It emerged from the context of Hilbert's formalist-finitist-axiomatic programme, but to an important extent broke away from this programme. However, to describe the text as a junction along this narrative requires a broader context than just the text of the proof. I would like therefore to describe Gödel's text as a point of diffraction and strategic choices on a smaller scale relevant to this twice-written text. The purpose of this brief discussion is to bring to light the many conflicting reader positions Gödel negotiated with, a plurality which is an important constituent of the plurality of the text itself.

The first and most obviously stated issue is that of self reference. **The solution suggested by Whitehead and Russell, that a proposition cannot say something about itself, is too drastic. We saw that we can construct propositions which make statements about themselves** (1934, 362). The narrative described above, that of **so to speak by chance** discovery, whereby a certain given formula stated that some well-defined formula was unprovable, and then, subsequently, it turned out that both formulas were one and the same — this narrative is the edifice meant to support the reintroduction of self reference into logic. There is also syntactic support for allowing such formulas into logic. On the same page we are reminded that **in fact, these are arithmetic propositions which involve only recursively defined**[28] **functions, and therefore are undoubtedly meaningful statements**.

[28] The term **recursively defined** refers to a formal property that includes being computable by a finite mechanical procedure.

This syntactic status of Gödel's self-referential propositions guarantees that Hilbert and the intuitionists accept the proposition, whereas the revisionist narrative confronts the possible objections of more logicist thinkers such as Russell, concerned with impredicativity.[29] Gödel manages to force self-reference into logic by having it comply with the standards of those who may consider self-reference objectionable.

But there remains still a position, an ascetic formalist position, which will accept the formula, but decline the suggestion that it is self referential. It will agree that the formula (as well as its negation) is unprovable, but refuse to accept that it **says** that any formula, and in particular itself, is unprovable. The text makes an effort to accommodate this position as well. Various footnotes already quoted above suggest that the text is not *the* proof, but an indication of how to carry it out, and that whatever is constructed is not the unprovable and irrefutable proposition, but only indicates this proposition (e.g. **Note that "**$[R(q);q]$**"** ... **is merely a metamathematical description of the undecidable proposition. But, as soon as the formula** S **has been obtained, we can, of course, also determine the number** q **and, therewith, actually write down the undecidable proposition itself** (1931, 149)).

At this point the readers are required to choose whether they consider the text to be a proof, or only an explanation of how to construct the proof; whether this text stands by itself, or hovers over some unwritten — but writable — formal text; whether one takes the **mechanical** stance attributed by Gödel to all his predecessor formalists,[30] or whether one allows the text to signify, sometimes even **so to speak by chance**. It seems that the text supports both positions at once. On the one hand it deploys formal **Theorem–Proof** arrays, and on the other hand it keeps recalling the writable, but unwritten, accompanying text. This choice (or rather refusal to choose) is related to another point of diffraction, that which tries to regulate the relations between syntax and semantics.

The sketch of the argument offered in the 1931 introduction works only provided we concede to the hypotheses articulated in the following quota-

[29] Impredicativity is a certain form of circularity, which was a major theme of concern in the context of Russell's critique on Frege's logic.

[30] See the fourth subsection of the previous section of this chapter.

tion. **The method of proof just explained can clearly be applied to any formal system that, first, when interpreted as representing a system of notions and propositions, has at its disposal sufficient means of expression to define the notions occurring in the argument above (in particular, the notion "provable formula") and in which, second, every provable formula is true in the interpretation considered**. The second hypothesis refers to **truth**, which the formalist tradition attempts to exclude from mathematical argumentation in favour of the notion of formal provability. **The purpose of carrying out the above proof with full precision in what follows is, among other things, to replace the second of the assumptions just mentioned by a purely formal and much weaker one** (1931, 151). The text promises to rise to the challenge of setting **truth** aside. But the immediately following statement suddenly reintroduces semantics into the picture. **From the remark that** $[R(q);q]$ **says about itself that it is not provable, it follows at once that** $[R(q);q]$ **is true, for** $[R(q);q]$ **is indeed unprovable (being undecidable). Thus, the proposition that is undecidable in the system *PM* still was decided by metamathematical considerations**. The text is willing to adopt strict formalist and intuitionist standards wherever it can, but not at the expense of doing away with points of view that accept **truth** as a legitimate notion. Gödel is well aware that this part of the argument will not be universally accepted; but he chooses not to suppress it.

The text does indeed stand at a point of diffraction, but it refuses to make a choice. Gödel's own preferences were stated openly in various texts, and are reconstructible from various others, but it is quite striking to watch how the text refuses to yield both to Gödel's and to his opponent-readers' points of view.

For those readers who endorse a semantic and signifying outlook, there awaits another ingenious point of diffraction. Since self reference is allowed, the standard solution to the liar paradox is foreclosed. With self reference allowed one cannot argue that when I say that **I am lying**, the enunciating **I** and enunciated **I** are not the same **I** in order to avoid a paradox.[31] Instead of rearticulating the subject, another rearticulation is suggested. Let us

[31] Such as Lacan, among others, does in Lacan (1978, 139).

quote the immaculately phrased argument.

Suppose that on 4 May 1934, A **makes the single statement, "Every statement which** A **makes on 4 May 1934 is false." This statement clearly cannot be true. Also it cannot be false, since the only way for it to be false is for** A **to have made a true statement in the time specified and in that time he made only the single statement**. Gödel's solution relies on his claim that A **must specify a language** B **and say that every statement that he made in the given time was a false statement in** B**. But "false statement in** B**" cannot be expressed in** B (for reasons that Gödel had already presented by this point, but which we shall not include here), **and so his statement was in some other language, and the paradox disappears. The paradox can be considered as a proof that "false statement in** B**" cannot be expressed in** B (1934, 362–363).

Rather than split the subject into enunciating and enunciated, the language is split into the language of enunciation and the object language. These two languages cannot coincide according to the text, because otherwise a paradox will ensue. Gödel's solution to the paradox is stipulating that one cannot express in any properly specified language B the notion '**false** (or, for that matter, **true**) in B'. This solution is a combination of *segregation* and *censorship*. First, one segregates the different languages we speak. Then one censors any reference in a given language to its own notion of truth.

This rearticulation of languages and repositioning of truth will be analysed in the final chapter of this book from the point of view of Deleuze's conception of language. Here I would like to conclude by pointing out that this solution is not at all forced upon the text; that there is here a strategic choice, which might be interesting to study from an archaeological point of view. The list below is intended only to validate the existence of other possibilities that merit consideration. It does not presume to be exhaustive.

1. A solution adopted by the most radical formalists and intuitionists, consists of giving up the notion of truth altogether. We simply no longer assume that there is such a well defined complete notion, and settle for notions of provability. A somewhat different approach is to restrict the use of the term 'truth'. This direction is related to

Wittgenstein's analysis of Gödel's semantic argument (Wittgenstein 1978, 118–122).

2. Wittgenstein suggests (in the context of Russell's antinomy in Frege's set theoretic calculus) to simply not carry out the paradoxical argument. He concedes that it is possible to carry it out, but objects that it is useless, and is therefore simply better ignored, as if it was never discovered. **If you based something on** Frege's **system, I don't see that it would necessarily be detrimental if there were a contradiction in it, as long as this contradiction is just not used as a thoroughfare or a circus. Then this calculus fulfils its particular purpose. — This calculus might be used (a) to base something on (as Frege does), or (b) to calculate with (as nobody ever does). So we might say: Everything in the calculus works all right as long as we do not pass through the contradiction** (Wittgenstein 1975, 227). Contemporary research mathematicians sometime express similar views concerning the interface between set theory and category theory: **At the moment the situation is not unlike the one prevailing in the 18th century in the infinitesimal calculus** ... **we know about the dangerous spots, where not to swim, and try to stay away while continuing our exploration**.[32]

3. We can also adopt a radical finitist view, and deny the existence of an infinite sequence of natural numbers (see Rotman (1993) for such an option and for references to similar approaches). Another radical finitist approach may be adopted, which denies the unlimited recursive use of rules of inference. If the length of a proof is limited in advance, some contradictions may be prevented. To a certain extent this possibility is taken up by the study of the complexity of proofs, which was suggested in a letter sent by Gödel to von Neumann (Gödel 1986–2003, Vol. V, 373–377).

[32] This statement appeared in a printed abstract of Pierre Cartier's talk 'Living in a contradictory world: categories vs. sets', which took place in the 'Trends in the mathematical representation of space' conference in Boston University, December 1, 2007.

4. We may, as was (to some extent) suggested by Gödel and attempted by several authors (Gödel 1986–2003, Vol. I, 139), no longer subject formal systems to finite mechanical procedures of verification.

5. Another avenue to explore consists of giving up the unrestricted use of negation or the universal quantifier. If we deny the possibility of attaching them to just any formula, Gödel's construction might be avoided (I could not find a reference for such a strategy, but it would not surprise me to find that such a strategy exists in the literature).

6. Finally, we may allow the paradox to remain within our formal system, but attempt to contain it by adapting the rules of inference in such a way that a contradiction does not immediately imply any formula in the system. Gödel himself encounters such a system in his review of a paper by Church (Gödel 1986–2003, Vol. I, 256–259). Today this direction is studied under the title of *relevance* or *paraconsistent* logics.

Each of these solutions carries its own implications. Some solutions were suggested briefly in the literature and never seriously investigated, others led to the establishment of 'deviant' systems of logic that are still actively researched. I will not attempt to analyse the epistemic and strategic implications of any of these solutions, as this is an entirely different project (Gödel's motivation for suggesting his solution is easy to derive from his platonic stance, but from the point of view of the larger research community the question is, I believe, largely open). The above list was included only to point out that Gödel's solution, largely accepted today, is but a strategic choice, and not an inevitability.

Chapter 2

From the Formation through to the Hymen

In this chapter we will follow formalists in reducing everything to the repetition of syntactic rules. But rather than rid ourselves of meaning we will see it proliferate into reflections on and across what can only eventually be determined as in between.

2.1 Where is the meaning of it all?

Let us almost-repeat the analysis of the subsection of the former chapter, whose title is almost-identical to the title above; but let us now employ a somewhat wider lens. A clip-on concept of meaning is most strikingly introduced by Gödel's declaration that while **a formal system consists only of symbols and mechanical rules relating to them, the meaning which we attach to the symbols is a leading principle in the setting up of the system** (1934, 349).

Three statements can be derived from this declaration. First, **meaning** precedes the **formal system**. Indeed, it was there already in its **setting up**. Second, the **formal system** does not contain **meaning**. Indeed, **a formal system consists only of symbols and mechanical rules**.

Third, **meaning** is something **we attach to the symbols**. This clip-off/clip-on portrayal of **meaning** echoes one of Derrida's **essential predicates in a minimal determination of the classical concept of writing ... a written sign carries with it a force that breaks with its context, that is, with the collectivity of presences organising the moment of its inscription** (Derrida 1988a, 9).

While meaning has been there since before the creation of the formal system, the formal system itself as a collection of symbols and rules has the force to break loose from the presence of that meaning, which underlies it. But wherein lies this **force**? Does it lie within the sign, within its form and structural deployment, or is this force in fact to be attributed to the active intervention of a writer/reader/user? Is **force** part of the metaphysics of the sign, or is it part of what Peirce or Wittgenstein would call its use? There is at least one piece of evidence that supports the second conjecture.

Now we turn to some considerations which for the present have nothing to do with a formal system (1934, 346) is the first sentence of the second section of the 1934 text. These nothing-to-do **considerations** are the definition of the technical notion of recursive functions (which we mentioned already several times, but do not intend to define in this essay). Despite having nothing to do, **for the present**, with formal systems, these **considerations** use formal notations. Despite having nothing to do, **for the present**, with formal systems, these **considerations** are carefully designed in order to be imported into a formal system. And despite having nothing to do, **for the present**, with formal systems, these **considerations** are indeed imported into a formal system in section 5 of the 1934 text.[1] But still, **for the present**, these **considerations** are independent. The conclusion is that it is not the formal similarity or the possible future application that clips meaning onto these **considerations**. It is the declaration that these are **considerations which for the present have nothing to do with a formal system**, which toggles their relation to formal systems off, and the subsequent argument, which toggles the relation

[1] The 1931 text similarly states on page 157 that **We now insert a parenthetic consideration that for the present has nothing to do with the formal system *P***. In this text it is theorem V that transports these considerations into the formal system.

between recursive functions and formal systems back on. The supplemental meaning is thus *bestowed* upon the text by an adjacent text.

But it is also important to mind the temporal adverbial **for the present**. This adverbial does not only mark that the text reserves the right to switch on and off a certain meaning or a certain some**thing to do** with formal systems. It also recalls, at the very moment when some**thing to do** is foreclosed, that this some**thing to do** can, at some none-present moment, be reaffirmed. This statement articulates what is now barred as something that may in fact be pertinent, provided we escape, as we may, the chronology of the text, and skip a few pages.

When introducing the transformation of symbols and formulas into numbers, Gödel states that **the meaning of symbols is immaterial, and it is desirable that it be forgotten** (1934, 355). This desired forgetfulness is obviously impossible. What worse way to induce oblivion than by explicitly willing it? This, like the **nothing to do** declaration, does not simply clip off a certain meaning, it clips it on-and-off. This link is presently off, while right now, before our very eyes, absently on. The different contexts, the different meanings, do not exclude each other completely. They coexist in a temporality where the present does not exclude the future and the past — a temporality, which should perhaps be encumbered with the phenomenological terms of *anticipation* and *retention*.

If Gödel can clip **meaning** on and off so arbitrarily it is because, for him, **Mathematical objects have an independent existence and reality analogous to that of physical objects. Mathematical statements refer to such a reality, and the question of their truth is determined by objective facts which are independent of our own thoughts and constructions. We may have no direct perception of underlying mathematical objects, just as with underlying physical objects, but — again by analogy — the existence of such is necessary to deduce immediate sense perceptions** ... **While mathematical objects and their properties may not be immediately accessible to us, mathematical intuition can be a source of genuine mathematical knowledge** (Gödel 1986–2003, Vol. I, 30–31). This reconstruction of Gödel's view by Solomon Feferman is akin to Frege's statement that **A third realm must be recognised. What belongs to**

this corresponds to ideas, in that it cannot be perceived by the senses, but with things, in that it needs no bearer to the contents of whose consciousness to belong. Thus the thought, for example, which we expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no bearer. It is not true for the first time when it is discovered, but is like a planet which, already before anyone has seen it, has been in interaction with other planets (Frege 1967, 29).

If meaning were indeed an ideal object of that sort, then clipping it onto and clipping it off of the text would be a perfectly legitimate manoeuvre. The text is a ladder in our ascent towards ideal objective meaning. Many different ladders can be used to that effect. The same ladder can lead us to many destinations. And a ladder can, of course, also be left unclimbed.

But if this image is to be accepted, and the ladder is but a vehicle to, and not the carrier of, meaning, then the crossing of the gap between text and meaning must be accounted for. I shall not attempt to account for this crossing (the way Gödel treats this problem is surveyed in the second section of the previous chapter); nor will I attempt to prove that the gap is unbridgeable, and that in fact it needn't be bridged, because, one may claim, there is no such gap, because **meaning** is nothing but our use of the text (the first of those tasks is the concern of the glorious platonist tradition, whereas the second task has been the focus of the pragmatist-analytic traditions stemming from Peirce and Wittgenstein). Nor will I attempt to suggest that meaning lies latent within the text, constricting its form, but leaving some freedom to decompose and recompose the pieces (this would be a structuralist approach). Here I will attempt to investigate the possibility that what is clipped on-and-off is not meanings to texts, but texts to other texts.[2] I will try to investigate to what extent such clip-art can produce the effect of meaning.

[2] I will wait until the next chapter to contemplate the *bodies*, which clip texts on and off.

2.2 Verisimilarity

We have not given up on the possibility that the attachment and detachment of meaning has to do with the structure of the sign itself, rather than with those operating the sign. But before we resuscitate this possibility, we must hold off the pretence that meaning is indeed so easy to clip on-and-off. If it were so easily clipped on-and-off, one could simply clip on to an arbitrary text such as $\exists x(x=0)$ the meaning 'both this statement and its negation are unprovable', and circumvent Gödel's tedious construction. Or, in a less caricatural design, if it were so easy to clip meaning on-and-off, we might simply enumerate the formulas of the formal system Principia Mathematica (*PM*) arbitrarily, assigning the number 10 to the formula that reads 'formula number 10 in *PM* is unprovable', thereby enabling the logic of the proof and generating an undecidable proposition.[3]

But there are two pertinent objections to such slight of hand. First, 'unprovable' is not part of the vocabulary of *PM*, and in order for the statement 'formula number 10 in *PM* is unprovable' to be assigned a number at all, this statement (and the notion of unprovability) must be expressed by the resources of that language (or whatever other formal system we may choose to work with).

Second, and more importantly, the way we express the statement 'formula number x is unprovable' in the language *PM* already depends on the assignment of numbers to formulas. Indeed, *PM* is concerned with numbers, and it is only after we have coded formulas by numbers that *PM* can refer to formulas at all, and in particular articulate their provability.

[3]Here is a sketch of the semantic argument that would apply in such a situation (where *provability* refers to provability in the restricted formal sense of *PM*): If formula numbered 10 were provable, then the formula 'formula number 10 in *PM* is unprovable' would be provable, and therefore formula number 10 would be unprovable — a contradiction. If, on the other hand, the negation of formula number 10 were provable, then 'formula number 10 in *PM* is provable' would be provable, which, according to the previous deduction results in a contradiction. Therefore neither formula number 10 nor its negation are provable; formula number 10 is undecidable. In particular, the claim that 'formula number 10 in *PM* is unprovable', since it has just been demonstrated to be unprovable, turns out to constitute a true claim (the truth of formula number 10 does not contradict its undecidability, because our demonstration that the formula is true does not constitute a proof in the restricted formal sense of *PM*).

Consequently, the formula expressing 'formula number 10 in *PM* is unprovable' can only be written *after* the assignment of numbers to formulas is effected, and *after* the term 'unprovable' is articulated in *PM*. As a result, once we have articulated a formula in *PM* meaning 'formula number 10 in *PM* is unprovable', this formula *already* has a number. If after the fact we change the system of formula enumeration so as to assign to that formula the number 10, the expression of 'unprovability' in *PM* (which depended on the original enumeration) might no longer express 'unprovability' with respect to the new enumeration, and our formula might no longer mean 'formula number 10 in *PM* is unprovable'.

Gödel's task, then, was to be able to present the following process. First, construct a system of formula enumeration; then, given that system of enumeration, to translate into the formal system the statement 'formula number x in *PM* is unprovable'; finally, to find a number g, such that the formula that means 'formula number g in *PM* is unprovable' indeed turns out to be assigned the number g. Here's the bottom line: the assignment of non-ordinary meaning to formulas turns out to be quite harshly constrained, for something that is supposed to be *arbitrary*.

In order to produce Gödel's effect of meaning, it is not enough to declaratively impose a certain meaning on a certain formula. The meaning-imposing-declarations must obey constraints of *verisimilarity*.[4] **The verisimilar**, explains Kristeva, **is an assembly (the symbolic gesture par excellence, cf. Greek *sumballein* = putting together) of two different discourses, of which one ... projects upon the other, which serves as its mirror, and identifies with it beyond difference** (Kristeva 1969, 212). In order for Gödel's enumeration to be acceptable, the elements of the meaning attachment mechanism must be considered as identical on some level. The elements that Gödel **identifies beyond difference** are numbers on the one hand and symbols of a formal system on the other. This identification is that which allows for the **isomorphic image of the**

[4]The notion of *vraisemblance* is developed by Kristeva in her early semiotic work to explain how a fictitious literary text produces a sense of truth and reality — how we come to accept the literary text as a valid source of reflection on the world, even though it is entirely made up. This notion has little to do with classical notions of *vraisemblance*, which refer to non-rigorous persuasion as preliminary for mathematical proof (Brian 1994, 60, 216)

system *PM* in the domain of arithmetic (1931, 147).[5]

However, such identification requires readers to operate discursive mechanisms that set aside any differences between symbols of a formal text and numbers, despite the fact that almost every participant in the various manifestations of academic mathematical discourse in the early 1930s would assert that there were some significant differences. It is the fact that such identification **beyond difference** was acceptable by enough leading participants in the mathematical discourse of the time, regardless of the acknowledged difference, which allowed for the effect of verisimilarity.[6] In order to effect verisimilarity, Kristeva explains, the **semantics of the verisimilar postulates a resemblance with the law of a given society at a given point of time and frames it within a historic present ... the semantics of the verisimilar requires a resemblance with the fundamental semantic units** that is established enough to cross the relevant discourse's threshold of replication. Only then does it **present itself as "outside time", "identification", "effectiveness", while being more profoundly and uniquely conforming (conformist) to a (discursive) order already there** (Kristeva 1969, 212–213). Verisimilarity is precisely the effect of **outside-time-effective-identification** based on a contingency of discourse.

[5]Kristeva writes **Having reserved to science the domain of veridicity, that absolute knowledge whose every enunciation is drenched** (*irriguée*), **secretes a domain of ambiguity, a yes-and-no in which truth is a present memory (a secondary presence, but still there), phantom-like and originary: it's the extra-veridical domain of sense as verisimilar** (Kristeva 1969, 211–212). In the course of my quotations mathematical discourse is placed in the position of the verisimilar (literary, extra-veridical), rather than the scientific (true). This manipulation entails (or depends on) either a renunciation of certain privileges of the scientific discourse, or the marking of mathematical discourse as a fiction beyond science. For the current context, however, I need not justify either.

[6]The point here is not merely historical. The contemporary reader too must make a similar identification **beyond difference**. We could indeed imagine a future reader for whom this specific difference would be completely crossed out. My belief, however, which can only be demonstrated by a text-by-text analysis, is that any reader would have to identify **beyond difference** some discursive strata, or else end up with no meaning at all. My motivation in stating this belief is ethical. This belief has to do with assigning authority to texts and taking responsibility for their interpretation. This ethical dimension will be discussed more explicitly at the end of this chapter.

In order for Gödel's transcription mechanism, which turns formulas into numbers, to be acceptable (to appear 'real', to be verisimilar), it must bind different semantic units to each other. But it is not only a question of semantics. In order to achieve verisimilarity, it must also verisimulate a syntax. ***The syntactic verisimilar*** **would be the principle of** ***derivability*** **(of different parts of a concrete discourse) from the global formal system. A discourse is syntactically verisimilar if one can derive each of its sequences from the structured totality that this discourse is ... The semantic procedure of assembling together two incompatible entities (the semantic verisimulation) having provided the "effect of resemblance", it is now a question of verisimulating the very process that leads to this effect. The syntax of the verisimilar takes charge of this task** (Kristeva 1969, 213–214). The reader **recognises beyond the logical grid, which is that of an informative statement, an "object" whose "truth" is tolerable thanks to its conformity with the grammatical norm** (Kristeva 1969, 230).

It is remarkably easy to manipulate Kristeva's concept of verisimilarity, arising from a comparison of fiction literature and scientific texts, into the verisimilarity that Gödel's text must deploy in order to be able to clip its meanings on-and-off. Semantic units are coupled together: numbers are coupled to primitive signs, numbers are coupled to formulas, numbers are coupled to their representations inside a formal system (sequences of primitive signs), arithmetic functions are coupled to formal functional expressions, and metamathematical notions are coupled to arithmetic functions. But it is crucial to note that what reigns over these couplings is a rigorous **constructive** syntactic edifice. Only by submitting to these heavy constraints could the texts under our study announce and/or put in abeyance meanings of formulas and arithmetic expressions.

The role of verisimilarity in Gödel's text is clearly indicated in the representation of metamathematical operations by arithmetic ones. Recall that, when introducing the operation $x * y$, after it is defined formally/arithmetically, it is claimed that $x * y$ is the number of the formula obtained by concatenating formula number x and formula number y; but recall also that no effort whatsoever was made to justify this claim (we

discussed this issue in subsection 1.2.2 above). Indeed, one cannot propose an arithmetic validation here, because concatenation of formulas is not an arithmetic operation. This representation of concatenation by an arithmetic operation is held as evident, and it is so held, because it is based on constructions analogous in their syntax. The syntactic analogy then allows for a semantic coupling. And this syntactically founded edifice of semantic coupling is taken to be sound without further scrutiny. One could, of course, generate a formal system that would deal with both sign-sequences and numerals in order to create a formal framework for discussing concatenations and the arithmetic operation $*$ at once, but Gödel shows no need to do so. One could test the analogy between concatenation and the operation $*$ empirically for some x's and y's, but, again, Gödel expresses no need to do so (possibly because he distinguishes arithmetic from any empirical counterpart).

The analogy between concatenation and the operation $*$ stands as it is. This syntactically founded edifice of semantic coupling is taken to be sound without further scrutiny. *But this is not a gap in the proof. This is what enables the truth.* This **"like" — *substitutive* preposition that allows to take one *for* the other** is the operator that holds together the complex and ambiguous relations of representation, expression, saying, signifying and meaning analysed in the previous chapter, whereby the **signifier designates at least two signifieds, the form indicates at least two contents, content supposes at least two interpretations, and thus to infinity, all verisimilar because placed together under the same signifier (or under the same form, or under the same content, and thus to infinity)**. But our aim is to go on and demonstrate that they tip **into vertigo: *the nebulosity of sense* in which the verisimilar speech (the sign) is eventually submerged** (Kristeva 1969, 221–222). Indeed, we saw how the transcription between formal, arithmetic and metamathematical texts advances indefinitely, that is, until the point where the text arbitrarily marks it as come to an end.[7]

[7]One may object that these issues are unique to Gödel's project as a metamathematical project. This observation is not entirely unfounded. However, the discrepancies between informal and formal versions of texts, semantic content that is lost in translation, meta-arguments based on similarity of technical arguments, and semantic coupling of different practices — all these phenomena reflect the issues raised above, and are part

What binds together the whirling metamathematical, arithmetic and formal texts is nothing but a common syntax and common terms,[8] a commonality most strikingly exemplified by the typographical rendering of the metamathematical-turned-arithmetical by small capitals in the 1931 text (a formula in *PM* may be unprovable, in which case the corresponding number is labelled UNPROVABLE); but it is even more strikingly exemplified by the typographical identity in the 1934 text (both formulas and corresponding numbers are said to be unprovable). But this commonality has its limits. Gödel's informal semantic argument (see the section *Gödel's argument in brief* in the introduction of this book) shows that consistency implies incompleteness. However, the formal translation only shows that a stronger property (ω-consistency) implies incompleteness.[9] Something is

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and parcel of contemporary 'standard' mathematics. The ways in which they contribute to mathematical semiosis must, however, be analysed on a text by text basis in order to properly reflect contextual contingencies.

[8]One may claim that these discursive strata are held together by some essential analogy. I will not comment on this claim, because this would take me too far off my line of thought, and because the texts we study do not suggest such a claim. But in order not to leave this possibility completely unchallenged, I will note that Wittgenstein has led a fierce onslaught against 'analogy' as presumed origin for mathematical validity, and views mathematical practice as a set of rules binding *different* practices by *declaring* them analogous — a declaration that is psychologically and practically constrained, but not constrained by mathematics or by an abstract notion of 'analogy'. Consider for instance Wittgenstein's comments on using the vertices of a pentagram to count to 10. **You might call it two ways of counting glued together. We could have had one way of counting by putting people on the crossing points of the pentagram and another way of counting by assigning numerals up to ten persons. What looks like counting, in the case of a pentagram, is a way of correlating these two ways of counting. [A rule is made]** (Wittgenstein 1975, 118). Consider also the following impressive dialogue, which starts with the words of Wittgenstein: **Suppose you had correlated cardinal numbers, and someone said, "now correlate *all* the cardinals to all the squares." Would you know what to do? Has it already been decided what we must call a one-one correlation of the cardinal numbers to another class? Or is it a matter of saying, "This technique we might call correlating the cardinals to the even numbers"? *Turing:* The order points in a certain direction, but leaves you a certain margin. *Wittgenstein:* Yes, but is it a mathematical margin or a psychological and practical margin? That is, would one say, "Oh no, no one would call this one-one correlation"? *Turing:* The latter *Wittgenstein:* Yes.—It is not a mathematical margin** (Wittgenstein 1975, 168).

[9]According to the semantic argument, the provability of a statement implies its truth

lost in translation. But even this loss-in-translation is not enough to invalidate informal assertions based on syntactic and semantic verisimilarity.

A structural approach would assume that semiotic systems have structures, which the researcher should discover and compare. Post-structural critique challenges this assumption. The extraction of structure from a system is no longer considered a discovery, but an act of discursively constrained gluing together of one system to another system, the latter system dubbed the former's structure. Post-structural critiques will further indicate that the structuring of semiotic systems can never be definitively settled, and that the means of comparing structures are contingent as well. It's the contingency of establishing an **isomorphic image of the system *PM* in the domain of arithmetic** that the notion of verisimilarity serves to bring up. I am not denying here the possibility of mechanically translating formulas into numerals. I am insisting here on the contingency of allowing such mechanical translation as a framework for doing mathematics. The contingency I am pointing out here is akin to the contingency that allows us to identify magnitudes and numbers — an identification, which classical Greek geometers were loath to endorse.

I do not appeal to the notion of verisimilarity to trivialise or make a caricature of the mathematical endeavour. There's nothing trivial, neither philosophically nor pragmatically, in the subjugation of the mathematical text to verisimulating constraints. A form of verisimilarity that is accepted by prevailing discursive standards is precisely what's lacking in the construction that arbitrarily assigns the number 10 to the statement 'formula number 10 in *PM* is unprovable'. And the concretely different discursive criteria for verisimilarity in formal mathematics and (some versions of) philosophical logic are precisely what allows the statement 'this statement is false' to serve as an object of study in the latter, but not in the former discursive field. Both fields, however, have earned their place in the production of human knowledge.

— in this particular argument the truth that some statement is unprovable. The syntactic argument replaces the passage through truth by a passage through the adequacy of the coding of metamathematical relations by formal expressions of arithmetic relations (1931, theorem V). In fact, both arguments obey distinct syntactic constraints, and both require attaching meaning to text, at least if they are meant to demonstrate the incompleteness of a formal system, rather than just a senseless formal statement.

And still, within certain historical limits, formal mathematics, philosophical logic, and literary fiction all allow themselves to be carried along by the **radical gesture of verisimilarity**, which is **a *putting together* of opposite units of meaning sufficient for leading (the impossible) to truth**; linking together provability and non-provability, linking together non-provability and truth,[10] such that **the two opposites (the same and the different ...) synthesise themselves in a *same*, which *is* still verisimilar** (Kristeva 1969, 219).

2.3 Elements of verisimilarity

Having introduced the language of verisimilarity into the texts under discussion, we must articulate the detail of how verisimilarity functions there. I will not attempt an exhaustive survey of all elements that contribute to an effect of verisimilarity in those texts, but only indicate some elements, which put mathematical verisimilarity in a wider context, and which will later implicate the texts we study in the processes of semiosis that I am fishing for.

In reading texts by Raymond Roussel, Kristeva uncovers **the structure of the canonical subject-predicate sentence as a principal syntactic rule of the verisimilar**. It is well evident that this structure and its recursive reiteration is indeed a principal syntactic rule in the production of common formal systems, including, of course, those to which Gödel refers. But I am more interested in what Kristeva finds in the **interior of this law**, where **various secondary syntactic figures of the verisimilar are recognisable, in which are included: repetition, doubling, enumeration** (Kristeva 1969, 230), and which give rise to the effects of ***linearity* (origin-destination), and *motivation* (syllogism)** triggered by the syntactic verisimulating machine (Kristeva 1969, 214).

Gödel's main tool is the enumeration of formulas in a formal system. Reading the work of Roussel, Kristeva writes that **it is enough that "absurd" facts be arranged in a sequence of enumerations so that absurdity is taken over by each element of the sequence, in order**

[10] Indeed, Gödel's semantic argument, as presented in the footnote on page 99, includes deducing unprovability from provability and truth from unprovability.

for that absurdity to become verisimilar due to its derivability from a given syntactic grid. As her analysis continues, it appears to become more and more directly applicable to Gödel's stratagem. **In the same way, the enumeration of signs which deceive and of false statements**, as are included in Gödel's enumeration, **is not unverisimilar; their *sequence*, as a syntactic ensemble of units derivable from each other, constitutes a verisimilar discourse** (Kristeva 1969, 233–234).

Consider the following taxonomy of animals. **(a) those that belong to the emperor; (b) embalmed ones; (c) those that are trained; (d) suckling pigs; (e) mermaids; (f) fabulous ones; (g) stray dogs; (h) those that are included in this classification; (i) those that tremble as if they were mad; (j) innumerable ones; (k) those drawn with a very fine camel's-hair brush; (l) etcetera; (m) those that have just broken the flower vase; (n) those that at a distance resemble flies** (Borges 1999, 231). If Gödel's enumeration appears less unmotivated and objectionable than the above taxonomy of animals, which Foucault quotes from Borges, who quotes it from Franz Kuhn, who is said to have quoted it from the **unknown (or false) Chinese encyclopaedia entitled 'The Celestial Empirium of Benevolent Knowledge'** (Foucault 1973, xv), it is because Gödel's enumeration follows a process, which sufficiently many participants in the relevant discourses recognise and replicate as a syntactic computational apparatus. Other than that, it is no more 'motivated' or 'justified' than John Wilkins' analytical language or the Aarne-Thompson system for classifying folktales, and yet it serves as an acceptable basis for analysis.

We are all familiar with the disconcerting effect of the proximity of extremes, or, quite simply, with the sudden vicinity of things that have no relation to each other; the mere act of enumeration that heaps them all together has a power of enchantment all its own: 'I am no longer hungry,' Eusthenes said. 'Until the morrow, safe from my saliva all the following shall be: Aspics, Acalephs, Acanthocephalates, Amoebocytes, Ammonites, Axolotls, Amblystomas, Aphislions, Anacondas, Ascarids, Amphisbaenas, Angleworms, Amphipods, Anaerobes, Annelids, An-

thozoans. ...' But all these worms and snakes, all these creatures redolent of decay and slime are slithering, like the syllables which designate them, in Eusthenes' saliva: that is where they all have their *common locus* ... startling though their propinquity may be, it is nevertheless warranted by that *and*, by that *in*, by that *on* whose solidity provides proof of the possibility of juxtaposition. It was certainly improbable that arachnids, ammonites, and annelids should one day mingle on Eusthenes' tongue, but, after all, that welcoming and voracious mouth certainly provided them with a feasible lodging, a roof under which to coexist (Foucault 1973, xvi).

Not so much unlike Eusthenes' list (borrowed by Foucault from Rabelais' *Gargantua and Pantagruel*), Gödel's enumeration allows to accept in bulk an entire sequence comprising provable and unprovable, true and false formulas. The sequence even allows sneaking in sequences of primitive signs that are not even syntactically correct formulas in the formal language under consideration. It creates a sense of homogeneity, which collects the sensible and the senseless into a common reservoir. **Invulnerable to all determined opposition between reason and unreason** (divisions of formulas into meaningful and meaningless, provable and unprovable, true and false) **it is the point starting from which the narrative of the determined forms of this opposition, this opened or broken-off dialogue** (between formal texts and meanings), **can appear as such and be stated**. The generation of this totality is the very gesture, which prescribes a position outside this totality (is this the position of meaning?). **It is the point at which the project of thinking this totality by escaping it is embedded. By escaping it: that is to say, by exceeding the totality**, by exceeding the formal system and attaining its meta-discourse and the discourse of its truth. Even if **nonmeaning has invaded the totality of the world, up to and including the very contents of my thought** ... even if **I do not *in fact* grasp the totality, if I neither understand nor embrace it, I still formulate the project of doing so** by presuming to enumerate everything, **and this project is meaningful in such a way that it can be defined only in relation to a precomprehension of the infinite and undetermined**

totality. I count, therefore I mean (Derrida 1978, 56, translation modified).

We must not forget, however, that enumeration is a form of repetition. In fact, repetition is a necessary condition for the entire syntactic edifice. It underlies not only counting and enumerating but also computing and the following of syntactic rules. Repetition appears in the texts under consideration not only through the interlingual transcription (the languages of the text, be they formal, arithmetical, metamathematical, or 'natural' are forced to repeat an articulation of the statement 'this statement is unprovable', each constrained by its own semantic units and syntax), but also through the very possibility of following syntactic rules. Syntactic rules are anchored to a line of repetition. One cannot establish syntactic coherence without an apparatus, which verifies that the statement to be examined *repeats* the syntactic rule. One cannot establish any form of analogy without recourse to the claim that one analogue repeats aspects of the other. To escape the fundamental and necessary condition of repetition, one must imagine a discourse that denies rituals, standards, and anything but self-validating epiphanic experience. It is tempting to conjecture that no such human discursive community can exist. It is certain that no such discursive community could *read* this text (although, perhaps, if we allow ourselves to wander into such incredibly far-fetched speculation, such a society could perhaps *consume* it). Repetition is the foundation of syntactic verisimulation, or at least it would be, if we could establish what repetition fundamentally is.

Kristeva considers repetition in modern literature as stemming from **the same obsession of repetition in the European literature of the late Middle Ages and the beginning of the Renaissance (the chronicles, the first novels in prose, the lives of saints, etc.)**. She continues with the following genealogy, citing Bakhtine. **Advanced research has demonstrated the vocal, phonetic and market-place origin of such enunciations: they arrive directly from the fair, the market, from the sonorous life of the commercial town or the army embarking on a journey. Hollered in full voice by merchants and heralds, the repetitive formulas** (*tournures*) **are the very kernels of a discursive practice generated within and for informing, and which is structured like a message, like a connection between**

a speaker and a recipient. They subsequently penetrate written texts (La Sale, Rabelais, etc.). In producing themselves at the very moment where the European structure escapes the domination of the symbol (Middle Ages) to submit itself to the authority of the sign (modernity), this phenomenon indicates once again the extent to which the structure of the verisimilar tale relies on the structure of phonetic communication (Kristeva 1969, 232-233).

Kristeva concludes her narrative with an analysis of her own object of study: Raymond Roussel's threefold *Impressions d'Afrique*. But to an extent that we must still investigate, this narrative seems applicable to the texts under our own analysis as well. A verisimulation of information and truth is generated by the same pseudo-informative repetition. The more carefully articulated, tediously minute, and syntactically computable the repetition — the more likely it is to appear verisimilar. The more languages it binds together, the more discursive strata it relates — the more likely it is to appear as a valid piece of information, consumed by an eager crowd in the market of scientifico-philosophical ideas. Within certain material limits, without repetition there is no verisimilarity.

If the function of the "sense" of discourse is a function of resemblance beyond difference ... **one could say that the *verisimilar*** ... **is a *second degree* of the symbolic relation of resemblance** (Kristeva 1969, 211). If we refuse to read this ***second degree*** as secondary with respect to the primal truth of science, then discursive verisimilarity would be secondary with respect to a radical repetition, the primitive manoeuvre that imposes the relations of *repetition* and *similarity* on distinctly different material entities (e.g. the word 'it' that has just appeared, and the word to appear next: 'it'; the application of an inference rule, and its next application; the formulation of an inference rule, and its application). This *repetition* is a euphemism for controlled difference, or perhaps for would-be-controlled difference, a difference that we wish-to-control with our will-to-power (the Nietzchean concept which Deleuze reads in his *Logic of Sense* as a will-to-elevate-to-the-n^{th}-power, a will-to-repeat).

If the verisimilar is a second degree of resemblance, but not with respect to a true resemblance of the first degree, then the verisimilar is secondary to a primary repetition, which primality, as we shall endeav-

our to see, is analogous to the primality of Freudian primary repression, namely a primality that is never actually present but always-already there. This is why **the verisimilar is an *effect*, a result, a product that forgets the artifice of its production**. This is also why the ***textual productivity*** that underlies the verisimilar ***is the inherent measure of the*** mathematical ***(of the text), but it is not the*** mathematical ***(the text)*, in the same way that all work is the inherent measure of value without being the value itself** (Kristeva 1969, 213, 238–239). Our goal is to observe this **inherent measure** of **text**, the processes that produce semiosis, that produce verisimilarity, that generate the function of sense or meaning as **resemblance beyond difference**, a process whose articulation Kristeva attributes to Jacques Derrida.

2.4 Iteration

For there is no word, nor in general a sign, which is not constituted by the possibility of repeating itself. A sign which does not repeat itself, which is not already divided by repetition in its "first time", is not a sign (Derrida 1978, 246). For for the sign, repetition is seminal (or rather *disseminal*, borrowing Derrida's term which we shall explore below). If something is not repeated and quoted, it does not function as a sign, but remains a plain object. For meaning, unlike communication as theorised in information theory, is not the effect of an Alice transmitting a sequence of bits to a Bob, a Bob whose sole role is to consume and make vanish the message, a Bob whose sole role is to restore the quiet and peace.

Meaning requires verisimilarity, and verisimilarity requires repetition. There is no meaning unless (to borrow again the imagery of information theory) there is an Eve to eavesdrop on the message — to eavesdrop means here to transcribe the text: to intercept it, to use it (meaning, after all, *is* use), to repeat it, to verisimulate with it. And since it is Eve who verisimulates meaning, Alice and Bob are discardable (but we do not discard the possibility that, on occasion, Eve happens to coincide with either Alice or Bob). **In order for my "written communication" to retain its function as writing, i.e., its readability, it must remain readable**

despite the absolute disappearance of any receiver, determined in general. My communication must be repeatable — iterable — in the absolute absence of the receiver or of any empirically determinable collectivity of receivers. This iterability (*iter*, once again, comes from *itara, other* in Sanskrit, and everything that follows can be read as the working out of the logic that ties repetition to alterity), structures the mark of writing itself, no matter what particular type of writing is involved ... **A writing that was not structurally readable — iterable — beyond the death of the addressee would not be writing** (Derrida 1988a, 7).

Alice and Bob may be dead (dead like Frege, dead like Russell, like Hilbert, or, perhaps, dead like I will someday be; but also, dead like the writer of the bulk of this essay already is when I insert this annotation). **Imagine a writing whose code would be so idiomatic as to be established and known, as secret cipher, by only two "subjects." Could we maintain that, following the death of the receiver, or even both partners, the mark left by one of them is still writing? Yes, to the extent that, organised by a code, even an unknown and nonlinguistic one, it is constituted in its identity as a mark by its iterability, in the absence of such and such a person, and hence ultimately of every empirically determinable "subject." This implies that there is no such thing as a code — organon of iterability — which could be structurally secret. The possibility of repeating and thus of identifying the marks is implicit in every code, making of it a network [*une grille*] that is communicable, transmittable, decipherable, iterable for a third, and hence for any possible user in general** (Derrida 1988a, 7–8).

The letters of the dead Frege, Russell and Hilbert (the last two still alive at the time) were intercepted, reiterated, verisimulated, resignified by Gödel. And by Derrida, and by us, you and I, reading this text. But, as we have already seen, criteria of verisimilarity vary from discourse to discourse. And the sign, intercepted, does not have the capacity to carry its context and rules of formation along with it. The text is therefore structurally open to other verisimulating, unanticipated meaning-generating transcriptions. Derrida's concept of the **decipherable** is a re-encoding of the terms

transmission and **iter**ation. For Derrida, to **decipher** the text, which remains beyond its generator and addressee, we need only repeat it, and, perhaps, verisimulate with it.

This is not only Derrida's metaphysical stance. This is also what Gödel does. He verisimulates with signs to the point of deciphering in them a vast rearticulation of fundamental concepts such as provability and truth. This is how he uses the text. Land, the communists used to say, belongs to whoever provides it with water. Text, as Gödel uses it, belongs to whoever repeats it verisimilarly.

2.5 Dangerous shifts of meaning

Actually, the process whereby material mathematics is put into formal-logical form, where expanded formal logic is made self-sufficient as pure analysis or theory of manifolds, is perfectly legitimate, indeed necessary; the same is true of the technisation which from time to time completely loses itself in merely technical thinking. But all this can and must be a method which is understood and practiced in a fully conscious way. It can be this, however, only if care is taken to avoid dangerous shifts of meaning by keeping always immediately in mind the original bestowal of meaning upon the method, through which it has the sense of achieving knowledge about the world. Even more, it must be freed of the character of an unquestioned tradition which, from the first invention of the new idea and method, allowed elements of obscurity to flow into its meaning (Husserl 1970b, 47).

Derrida reasserts the link between Husserl's reproach above to Plato's stance. **The advancement of science can be pursued**, says Derrida's Husserl, **even when the sense of its origin has been lost. But then the very logicality of the scientific gesture, imprisoned in mediacy, breaks down into a sort of oneiric and inhuman absurdity. Did not Plato describe this situation?** ... **"Geometry and the studies [*sciences*] that accompany it" are exiled far from their fundamental intuitions. They are incapable of "vision" (*idein*) and riveted to the hypotheses held as their principles. Confusing symbol with**

truth, they seem to us to dream ***(orōmen ōs oneirottousi)*** **(*Republic* VII, 533c)** (Derrida 1989, 107).

But this position is not restricted to old masters. I believe that this position underlies Harvey Friedman's motivation for seeking to replace Gödel's theorem by a **quest for a simple meaningful finite mathematical theorem that can only be proved by going beyond the usual axioms for mathematics** (Friedman 1998, 805). Could it be that syntactic verisimilarity is not enough for a statement to have genuine meaning? Could we avoid **dangerous shifts of meaning**?

Leaving this question aside until the end of this chapter, we note that **dangerous shifts of meaning** as articulated above are highly tangible in Gödel's text, especially, we observe again, in the gap between the so called 'semantic' and 'syntactic' arguments. The semantic argument presented in the 1931 introduction (see the section *Gödel's argument in brief* in the introduction above) proves that one must choose between undecidability and consistency. If we assume that the formal system is decidable, Gödel's argument shows that it cannot be consistent. However, the syntactic argument, which is supposed to transcribe the semantic argument inside the formal system, brings us to a slightly different conclusion: if we assume that the formal system is decidable, then we conclude that it is only ω-inconsistent, a property which is weaker, and perhaps less objectionable, than inconsistency.

The point is that the two linguistic strata end up diverging beyond the common meaning that is supposed to be their leading principle and make them reflect each other. This divergence has led to a state of affairs where some mathematicians accept the semantic argument, while others do not. This does not create a crisis in mathematics, because almost all mathematicians consider the discrepancy to depend on the syntactic and semantic framework inside which one works, and are clear on which framework allows which argument. Which framework, if any, is actually true, and in what sense of the term true — these are considered extra mathematical questions.

A tradition of mathematicians, which has become dominant (at least as far as most philosophers are concerned) has been developing a particular discursive strategy since the mid 19^{th} century, which became fully opera-

tional at the beginning of the 20^{th} century under the influence of the Hilbert and Bourbaki schools. While mathematical production is constrained and manipulated by many mechanisms, a mathematical argument's validity depends heavily on criteria of verifiable syntactic verisimilarity.

I do not mean to exaggerate the role of syntactic verisimilarity criteria. No mathematician has ever translated any but the simplest and shortest proofs into a formal text. The enterprise of automatic, computerised proof checking (e.g. Cantone et al. 2004) hardly ever finds mathematical application, not only due to technical difficulties, but also due to lack of interest by the mathematical community. There are even ways to discredit a mathematical argument without indicating a syntactic error (for instance, showing it to be inconsistent with other accepted results). But a mathematical debate concerning a suggested argument is not considered completely settled until a consensus is established concerning a formal error (which need not be identified at the most 'elementary' formal level, as such level of formalisation is practically never reached), or until the critics of the argument withdraw their claims for such error. Note, however, that in pointing out formal errors, there remains some room for debating the manner of formally transcribing an argument that best captures the argument's intention.

In their quest for consensus, substantial tracts of mainstream mathematical discourse bestowed upon syntax the power of final arbitration. And in doing so mathematical discourse has given up protecting itself against those supposedly **dangerous shifts of meaning**, which Husserl was worried about. The rules of mathematical syntax have changed, and may keep on changing. But at this historic moment, due to the strategy of relegating substantial authority to syntax, mathematics is one of the contemporary human discourses most exposed to the only partly controllable shifting (iteration, *différance*) of meaning. Many mathematicians embrace this fact, rather than oppose it. Today's mathematics will not have any substantial qualms with an equivalent of Bombelli's 'sophistic' techniques (the introduction of computation with roots of negative numbers) or of the violation of Euclid's fifth axiom, as long as they are syntactically verisimilar.

To the verifier of mathematical validity, **Meaning**, as Gödel writes, **is immaterial**. It's not that mathematicians don't debate questions of meaning (we have quoted Friedman above who does). It is that mathematicians

are not required to agree on issues of meaning in order to maintain collaborative mathematical production. Mathematicians should agree on priorities (otherwise, they cease to communicate, resulting in the deep fragmentation of mathematics today, which makes it very hard for specialists in different fields to understand each other). Mathematicians should agree on syntax (where syntax should be understood as rules of repetition and recursion that, under current material and historic conditions, generate consensus; Wittgenstein devoted hundreds of paragraphs to convince philosophers that nothing more 'essential' can underlie the following of (syntactic) rules). Where it can be decided that a sign has escaped mathematical syntax, there an outside of mathematics can be warded. Where the relations between a mathematical sign and mathematical syntax are undecidable, there the limits of mathematics are undecidable as well.[11]

Due to this concrete and historic contingency of mathematics, post-structural conceptions of semiosis are in a way easier to establish in mathematical discourse than in other discourses. The mathematical sign, more obviously than any other sign, is thoroughly exposed to supposedly **dangerous shifts of meaning**. I will show how such shifts operate inside Gödel's proof in the section following the next.

But what is this supposed **danger**, which I insist on welcoming into mathematics? Husserl's **danger** is obviously not that of a formal contradiction. I do not claim that shifts of meaning will necessarily entail a formal collapse of logical systems. The **danger** is that meanings associated with the motion of mathematical signs will run amok, and lose their **original** grounding. Such danger is indeed prevalent in mathematical discourse: new meaning formations may not only diverge from **original** ones, but may even prove to be semantically contradictory.

A classic example is that of the square root of -1. One can prove that such an object does not exist. But the proof does not prevent the introduction of this very object into mathematics. To avoid a *formal* contradiction, the non-existence of a square root of -1 is rearticulated as the non-existence of a *real* square root of -1. Ridding mathematical structures

[11]Such as in the joke $\frac{\cancel{6}4}{1\cancel{6}} = 4$, which despite the correct result is a formal mistake, but at the same time could be considered as a correct formal manipulation under certain restrictive conditions to be explicitly specified.

of formal contradiction is not a difficult task for a proficient logician. But during this manoeuvre to escape formal contradiction, the term 'number' too is irreducibly displaced away from its **origin**.

Such processes prevail throughout mathematics. The transformations of the term 'line', for example, as it went through non-Euclidean and algebraised geometry, form a geometric analogue of the transformations that afflicted the concept of number through the introduction of negative and complex numbers. This process still goes on today. The recently defined notion of *noncommutative geometry* is a contemporary example.[12]

But, again, why is all this so **dangerous**? After all, we know well that one can, a-posteriori, look back and articulate a common 'essence' shared by the entire genealogy of notions such as 'number', 'line' or 'geometry' (or, at least, by those components of the genealogy deemed relevant for the extractor of 'essence'). The **danger** is that such 'essences' fail to be **original** in a phenomenological sense, yielding the equivalent of applying propositional logical analysis to a sentence that's senseless as to its content, such as **the sum of the angles of a triangle is equal to the colour red** (Husserl 1969, 220), which, according to Husserl, renders the law of excluded middle defunct. Rearticulated meanings of mathematical terms may result from the motion of signs and from the narrative ingenuity of the constructors of post-hoc meaning, dangerously imposing established rules on objects that exceed their scope of application.

Nevertheless, even if the phenomenological grounding of meanings remain obscure for author and readers alike, no referee will complain that a submitted proof is formally sound, but unacceptable because the **original bestowal of meaning** has been given up (the referee may complain that the result is irrelevant or uninteresting, or protest against a certain terminology). Contemporary mathematical discourse simply does not require the establishment of an adherence to an **original bestowal of meaning**.

And still, to properly assess Husserl's danger we must ask what is this phenomenological **origin**? Is it some early 20^{th} century axiomatisa-

[12]The study of geometric objects through associated algebraic structures has been abstracted to the point where the resulting generalised algebras may no longer have any underlying geometric objects. These algebraic objects, however, are still studied using geometric terms and motivations.

tion? Does it perhaps lurk inside Euclid's *Elements*? Or is the **origin** the first historic instance of number ever to appear? For Husserl the **origin** relates to historicity, but is not assignable a concrete moment in linear time. Husserl's origin is the very phenomenological inauguration of mathematical reasoning. Derrida reads into Husserl's *Origin of Geometry* that, according to Husserl himself, this inauguration is none other than an openness to unanticipated articulations of meaning. More precisely, **starting from this inaugural infinitisation** (Greek mathematics as an infinitely open potentiality for the production of theorems within a *fixed* axiomatisation) **mathematics cognises new infinitisations** (axiomatisations) **which are so many interior revolutions** (Derrida 1989, 127). Only if the **origin** is understood as openness to revolution, says Derrida's Husserl, can we remain committed to it. But such commitment is not dominated by any present meaning, cannot be fettered to any platonic determination, and challenges any attempt to confine rules to specific realms of objects (by performing the mathematical equivalent of applying propositional logic to statements that are senseless as to their content, we may be committing a fallacy, but we may also inaugurate new strata of sense, whose use is not subordinate to attempted 'reactionary' phenomenological analyses of origins). If this openness is the **origin** we must adhere to, then this **origin** is precisely the inaugural submission to **dangerous shifts of meaning**. And therefore, from the point of view of post-structural thinking, this is not so much a **danger**, as a constitutive condition for semiosis as such.

2.6 *Omne symbolum de symbolo*

The picture painted above may be considered nonsensical, unscientific, counter-productive. Before I go on to establish it at the micro-level of Gödel's proof, I wish to indicate that this position is actually quite pragmatic, at least in the sense of invoking some thinkers subsumed under that term.

The picture above may appear awkward, since we are used to expect that ***mimésis*** of truth by a text **has to follow the process of truth ... its order, its law**; since it is **in the name of truth, its only reference — *reference* itself — that *mimésis* is judged, proscribed**

or prescribed according to a regular alternation. In Gödel's texts, however, which are subject to the authority of verisimulating syntax and of repetition (which bring about dangerous shifts of meaning) **reference is discretely but absolutely displaced in the workings of a certain syntax, whenever any writing both marks and goes back over its mark with an undecidable stroke**. Marking here is writing along with syntax; going back over the mark is syntax's undermining of the act of reference governed by truth. Marking lays down discursive strata; at the same time, having forsaken the providence of referential truth in favour of the rules of syntax, marking conflates the discursive strata into a complex mesh of inter-representations. The undecidability here is of course not simply that of Gödel's formula, but that of the complex and unstable mesh of inter-representation. **This double mark escapes the pertinence or authority of truth: it does not overturn it but rather inscribes it within its play as one of its functions and parts** (Derrida 1993, 193).

When read as part of Derrida's metaphysical intervention, such subjection of truth to formations of repetition tends to be viewed as highly disconcerting. However, in the presence of Gödel's rearticulation of truth as at once presiding over each and every language, and at the same time necessarily subject to the confines of the syntax of some (meta) language,[13] Derrida's formulation loses much of its mystery.[14] In fact, one can summarise this reading by combining the voices of Peirce and Derrida, as in the following passage from *Of Grammatology* (Derrida 1976, 48–50).

> **Symbols grow. They come into being by development from other signs, particularly from icons, or from mixed signs partaking of the nature of icons and symbols. We think only in signs. These mental signs are of mixed nature; the symbol parts of them are called concepts. So it is only out of symbols that a new symbol can grow. *Omne symbolum de symbolo*** (Peirce 1931–1958, Vol. 2, §302).

[13] See section 7 of the 1934 text, where Gödel follows Carnap and Tarski. This is where Gödel claims that the truth of a given language cannot be expressed in that language, as surveyed in the last section of the first chapter.

[14] In the next section I will attempt to restore some of this challenging mystery by applying it not to the rearticulation of the concept of truth, but rather to the operation of substitution and to the **hymen** which allows it.

Peirce complies with two apparently incomparable exigencies. The mistake here would be to sacrifice one for the other. It must be recognised that the symbolic (in Peirce's sense the Saussurian **"arbitrariness of the sign") is rooted in the non symbolic, in an anterior and related order of signification: "Symbols grow. They come into being by development from other signs, particularly from icons, or from mixed signs." But these roots must not compromise the structural originality of the field of symbols, the anatomy of a domain, a production, and a play: "So it is only out of symbols that a new symbol can grow.** ***Omne symbolum de symbolo.*****"**

But in both cases, the genetic root-system refers from sign to sign. No ground of nonsignification — understood as insignificance or an intuition of a present truth — stretches out to give it foundation under the play and the coming into being of signs. Logic, according to Peirce, is only a semiotic: "Logic, in its general sense, is, as I believe I have shown, only another name for semiotics (semeiotike), the quasi-necessary, or formal, doctrine of signs." And logic in the classical sense, logic "properly speaking," nonformal logic commanded by the value of truth, occupies in that semiotics only a determined and not a fundamental level ... **It is a matter of elaborating** ... **a formal doctrine of conditions which a discourse must satisfy, in order to have a sense, in order to "mean," even if it is false or contradictory. The general morphology of that meaning (*****Bedeutung, vouloir-dire*****) is independent of all logic of truth** ...

Peirce considers the indefiniteness of reference as the criterion that allows us to recognise that we are indeed dealing with a system of signs. What ***broaches the movement of signification is what makes its interruption impossible. The thing itself is a sign*** ... **According to the "phaneoroscopy" or "Phenomenology" of Peirce,** ***manifestation*** **itself does not reveal a presence, it makes a sign. One may read in the** ***Principle of Phenomenology*** **that "the idea of** ***manifestation*** **is the idea of a sign." There is thus no phenomenality reducing the sign or the representer so that**

the thing signified may be allowed to glow finally in the luminosity of its presence. The so-called "thing itself" is always already a representamen shielded from the simplicity of intuitive evidence. The representamen functions only by giving rise to an interpretant that itself becomes a sign and so on to infinity. The self-identity of the signified conceals itself unceasingly and is always on the move. The property of the representamen is to be itself and another, to be produced as a structure of reference, to be separated from itself. The property of the *representamen* is not to be *proper* [*propre*], that is to say absolutely proximate to itself (*propre*, proprius). *The represented* is always already a *representamen.* Definition of the sign:

Anything which determines something else (its interpretant) to refer to an object to which itself refers (its object) in the same way, this interpretant becoming in turn a sign, and so on ad infinitum. ... If the series of successive interpretants comes to an end, the sign is thereby rendered imperfect, at least (Peirce 1931–1958, vol. 2, §303).

Under the sovereignty of repetition and verisimulating syntax there is no division between 'motivated' sign (iconic, reflecting its reference) and 'unmotivated sign'. When recursive syntax is relegated authority, all we have is a becoming-unmotivated of what may be, on a different level, motivated. The syntax uproots the sign from whatever supposed ground allegedly generated it, and turns it into **trace**. **In fact, there is no unmotivated trace: the trace is indefinitely its own becoming-unmotivated** (Derrida 1976, 47). Yes, the sign did supposedly emerge historically from certain concrete practices, but it is not this supposed origin that rules over its use. It is rather syntax, revision, and unanticipated practices that appropriate the sign and manipulate it. This indefinite and disseminal motion of appropriation and rearticulation is that which Derrida calls **trace** (a theoretical model of this supposed *ground* and alleged *generation*, a debate which at once allows the sign to be materially generated and but always-already subject to **its own becoming unmotivated**, is deferred to the next chapter).

The long quotation above was meant to supply theoretic support to

the mesh of meanings in Gödel's text and to its rearticulation of the concept of truth (surveyed in sections 2 and 3 of the first chapter) as processes of submission to verisimulating syntax. In order to re-stitch this theoretical debate to the mathematical context, let's recapture another paragraph that Derrida quotes from Peirce (Peirce 1940, 99), one which I covered up with an ellipsis in the preceding quotation.

The science of semiotics, writes Peirce, borrowing from medieval thinkers, **has three branches. The first is called by Duns Scotus *grammatica speculativa*.[15] We may term it pure grammar. It has for its task to ascertain what must be true of the representamen used by every scientific intelligence in order that they may embody any meaning. The second is logic proper. It is the science of what is quasi-necessarily true of the representamina of any scientific intelligence in order that they may hold good of any object, that is, may be true. Or say, logic proper is the formal science of the conditions of the truth of representations. The third, in imitation of Kant's fashion of preserving old associations of words in finding nomenclature for new conceptions, I call *pure rhetoric*. Its task is to ascertain the laws by which in every scientific intelligence one sign gives birth to another, and especially one thought brings forth another**.

This trichotomy of the semiotic project recognises a stratum on which Gödel's text can articulate a grammar that precedes truth — this is the *grammatica speculativa*, which deals with the constitutive conditions for **embody**ing **meaning**, and which precedes logic as dealing with representamina **hold**ing **good** or **be**ing **true**. To an extent, this is Gödel's attempt when he articulates and enumerates all formulas in a formal system in total disregard of their potential meaninglessness and falseness. But perhaps Gödel's task is even more radical, because in this list he does away even with meaning (or at least one layer of meaning). This is a moment of the **desirable that it be forgotten**, where truth and meaning are cast away so as to allow a survey of the syntactic field of opportunities, and to allow this survey to carry the text adrift. Upon such foundation rest formal

[15] Derrida corrects the reference and attributes the *grammatica speculativa* to Thomas d'Erfurt.

systems (that is, perhaps, why formal systems often tolerate inference manoeuvres such as *reductio ad absurdum*, which allows folly and deceit to establish a confined madhouse in which to carry out their frolicking and deliver to the outside an antagonistic revelation of truth).

Husserl's **dangerous shifts of meaning**, the constant motion of becoming-unmotivated without an original ground to protect us against the motility of signs, do not arise only from the repetition inscribed in verisimulating syntax. These dangers arise also form a redirect link to the hubris of enumerating everything (all formulas of a formal language), of desiring to forget meanings, of presuming to control formulas of folly and deceit. For **even if the totality of what I think is imbued with falsehood and madness**, as in *reductio ad absurdum*, **even if the totality of the world does not exist**, as in the open hierarchy of languages and their truth predicates, **even if nonmeaning has invaded the totality of the world, up to and including the very contents of my thought**, because meaning has lapsed into an oblivion of the desire to forget, **I still think, I am *while* I think**, or if we are somewhat less metaphysically presumptuous, at the very least, I still mean. *I repeat, therefore I mean.*

But this crisis in which reason is madder than madness — for reason in its manifestation as syntactic verisimilarity **is nonmeaning and oblivion — and in which madness is more rational than reason, for it is closer to the wellspring of sense**, that phenomenological origin reformulated as an openness to shifts of meaning that veers between forgetting and recollecting, between clipping meaning on and off, **this crisis has already begun and is interminable** ... **And nowhere else and never before has the** manifestation **of *crisis* been able to enrich and reassemble all its potentialities, all the energy of its meaning, as much, perhaps, as in** Gödel's 1931 and 1934 texts (Derrida 1978, 56, 62).

2.7 ,

That the sign carries within itself the potential to escape and revolutionise its context, that the sign cannot be guarded against **dangerous shifts of meaning**, are claims that Derrida has insisted on rediscovering

across a myriad of semiotic and metaphysical approaches, arguably sampling the better part of western intellectual history. Without presuming to exhaust Derrida's analyses, I will proceed to comment, using his tools, on the mathematical-semiotic implications of relegating authority to syntactic verisimilarity and of the (dis)seminal privilege of repetition. My task is to demonstrate these implications not on a historic scale, but within the confines of the synchrony of a 'single' mathematical text — Gödel's proof. For those who follow Derrida, finding it all in a mathematical text is clearly to be expected — but such expectations make for a dangerous assumption of closure, until they challenge the reader in their own singular ways.

Up to this point we have synthesised a pragmatist-oriented reading of Derrida with our analysis of semiosis in Gödel's 1931 and 1934 texts. This synthesis, this quilt, is meant to re-cover mathematical discourse in its attempt to reign over meaning and truth while subjecting itself to the rules of verisimulating syntax. Reiteration is manifest in this project as a complex large scale reappropriative deciphering. This last phrase describes both Gödel's manoeuvre of rearticulating meaning and truth with respect to an entire (meaning/less, un/true) field of textual combinations (formulas), as well as this essay's feeding of the 1931 and 1934 texts into the work of Derrida and Kristeva. Such reappropriative manoeuvres needn't *maintain* meaning (*main-tenir*, grab by the hand, arrest). In such macroscopic assemblages **meaning** is indeed *repeated*, but only provided we keep in mind that repetition is **resemblance beyond difference**, and that **resemblance** is not strictly a precondition for repetition, but, in fact, conditions repetition as much as it arises as its effect.

This bird's-eye view is not enough. In order to imbue the mathematical text with the *rule* (both sovereign and decreed) of iteration, the effects elaborated above must intervene when the most 'simple' forms of repetition occur. To demonstrate the hold of the sign's motility on the mathematical text we must explicate how, from the very first moment I recognise a sign as a sign, I already *re*-cognise a sign (1) as a sign (2) — how, in the mathematical text, I recognise that a sign is open to repetition, which will resemble it beyond factual differences.

Consider, for instance, the elementary repetition quoted from the 1934 text: $S(z_p, z_p)$. We'll quickly review the **meaning** of this text. To keep

things containable, I will not repeat the explication of the many complex discursive interrelations and ambiguities involved in accounting for $S(z_p, z_p)$. However, it is desirable (I so desire) that this thick of meaning not be forgotten.[16]

First, we distinguish between *number* and *numeral*. A numeral is the **representation** of a number in the formal language. The number 3, for instance, will be **represented** by the numeral $N(N(N(0)))$ — which **means** 'the successor of the successor of the successor of zero'. Any other number x will be *similarly* **represented** by a sequence of x such N's (x iterations of the successor function). Since such strings *cannot be* (practically or essentially) *written* for very large constants and for variables, these strings are **denoted** by the compact text z_x. The numeral $N(N(N(0)))$, for instance, is thus **denoted** by z_3. z_x **denotes** a text in the formal system, but is not itself a text in the formal system.

The term $S(z_a, z_b)$ is a function, **expressed** in the formal language, which takes as input two numerals (denoted by z_a and z_b), and outputs a third numeral. This third numeral is obtainable in the following way:

1. Take the formula **represented** by the number a.
2. Substitute the numeral **denoted** by z_b for all free occurrences of the variable w in this formula (if the formula contains such occurrences).
3. Compute the number **representing** the resulting formula.
4. Output the numeral that **represents** this number.

If, for example, z_{11} had denoted the numeral representing the formula $w = 0$, then $S(z_{11}, z_3)$ would have been the numeral representing the formula $N(N(N(0))) = 0$.

It is not yet necessary to explain what the number p **means**. But note that if one considers the term $S(z_p, z_p)$, then the first z_p **represents**

[16]... **he disguises every proper name as a description and every description as a proper name, showing by way of this ruse, that such a possibility, always an open one, was constitutive of writing, to the extent that literature** (and mathematics) **works it over on all sides. You never know whether he names or describes, nor whether the thing he describes-names is the thing or the name, the common or proper name**.(Derrida 1984, 118).

a formula, whereas the second z_p **denotes** a numeral to be substituted into that formula. z_p is repeated, but its **meaning** is changed. One could, of course, in theory if not in practice, write down the formal expression for S, and carry out the argument with no reference to the meaning of z_p. In fact, in order to carry out the formal proof one needn't even recognise that the text denoted by z_p is a numeral. In fact one needn't even realise that one has proved undecidability at the end of the argument — one can, in principle, formally prove the statement without even realising that one has proved an arithmetical statement. But I will defer the analysis of (whatever is left of) semiosis in such a reading to the next chapter. For the time being we'll follow the texts under our analysis in embracing the discourse of meaning, and acknowledge the change in meaning of the repeated term.

This situation is, of course, not unique to the mathematical text. In poetic language **units are non-repeatable or, to put it otherwise, the repeated unit is not the same, so that one can guarantee that once repeated it is already another. The apparent repetition XX is not equivalent to X. There appears a phenomenon, unobservable on the (manifest) phonetic level of the poetic text, but which is a specifically poetic effect of sense, and which consists in reading within the (repeated) sequence both itself *and* something else** (Kristeva 1969, 259). Kristeva goes on to quote examples by Baudelaire, Mallarmé (**L'Azur! L'Azur!, L'Azur!, L'Azur!**) and Poe (**Nevermore**).

But in fact one does not even need to consider the poetic as special in this way, or pretend that the difference does not exist on the phonetic level. **For example, we may hear in the course of a lecture several repetitions of the word *Messieurs!* ('Gentlemen!'). We feel that in each case it is the same expression: and yet there are variations of delivery and intonation which give rise in the several instances to very noticeable phonic differences — differences as marked as those which in other cases serve to differentiate one word from another (e.g. *pomme* from *paume*, *goutte* from *goûte*, *fuir* from *fouir*, etc.). Furthermore, this feeling of identity persists in spite of the fact that from a semantic point of view too there is not absolute reduplication from one *Messieurs!* to the next** (Saussure

1966, 106–107). To that adds Oswald Ducrot that **it is evident that the simple repetition of the word conferred on its second appearance a nuance of irritation or of pleading absent from the first** (Wahl 1968, 45).

But so far, we have only observed a simple polysemy of the sign z_p: it could signify a number or a formula, depending on its syntactic place. This is not a very exciting observation, and not as radical as the difference that arises from the very fact of repetition. After all, if polysemy can be disambiguated by syntactic position, and the second z_p represents a numeral rather than a formula because it is differently syntactically placed, then the second z_p is not a proper repetition of the first at all (in the same way that the noun *stand* is not a proper repetition of the verb *stand*). Meaning, here, one may object, is not dangerously shifted by repetition; meaning may turn out to be well-defined and stable, if we consider the interaction between the *position* and the sign. Meaning could, perhaps, be held fast to its place.

But the delicate point here, as in the examples of Kristeva and de Saussure above, is not the discernible differences of intonation, phonic substance or syntactic and semantic role. What confers the nuance of irritation or pleading on the second ***Messieurs!*** is the very fact of repetition. The measurable phonic differences themselves do not lead de Saussure to consider the repeated term as an independent entity. The repeated term exists by definition only in so far as it is preceded by that which it repeats. If it is not so preceded, it is simply not a repeated term, regardless of its phonetic regularities or singularities. This is the sense in which the repeated term must be different from any non-repeated equivalent: the non-repeated term stands alone; the repeated one does not. Unlike the non-repeated term, the repeated term cannot be isolated and made to stand alone; if it were, it would no longer be a repeated term.

Indeed, the effect of repetition in our mathematical case, $S(z_p, z_p)$, is much more radical than a simple polysemy determined by syntactic position. In order to demonstrate this claim we must unveil the number p, and quickly review a portion of Gödel's argument. p is the number denoting the following formula: $\Pi v[\neg B(v, S(w, w))]$, where Π is the universal quantifier, $\neg$ is the negation sign, S is the functional expression defined above, and

B is a predicate in the formal system that represents provability ($B(z_a, z_b)$ holds if and only if the number a represents the proof of the formula represented by the number b). v and w are numeral variables. This formula reads that for any numeral that we may substitute for v, this numeral does not represent a proof of the formula represented by $S(w, w)$ — once w is substituted by a numeral. Since numerals cover all possible proofs, the formula represented by the number p reads: the formula represented by $S(w, w)$ (after substitution of a numeral for w) is unprovable. Note that whether that statement will turn out to hold or not, to be true or false, may depend on what we will substitute for w.

Let us now use the 1934 text's notation to recapture the argument from the introduction to the 1931 text. Let's also use the 1931 typographic convention, which substitutes for 'the numeral denoted by z_x represents a(n) (un)provable formula' the shorthand 'z_x is (UN)PROVABLE'.

According to the definition of S, the expression $S(z_p, z_q)$ represents the formula number p (the formula $\Pi v[\neg B(v, S(w, w))]$) with z_q substituted for the free variable w. $S(z_p, z_q)$ therefore represents the formula $\Pi v[\neg B(v, S(z_q, z_q))]$, which claims that $S(z_q, z_q)$ is UNPROVABLE. In other words, if $S(z_p, z_q)$ is PROVABLE, then the $S(z_q, z_q)$ is UNPROVABLE. This is so for any q. Now let's substitute p for q. We get that if $S(z_p, z_p)$ is PROVABLE, then $S(z_p, z_p)$ is UNPROVABLE, and end up in a contradiction.

We shall not continue here with Gödel to derive a contradiction from the possibility that the NEGATION of $S(z_p, z_p)$ is PROVABLE, and to conclude that $S(z_p, z_p)$ is UNDECIDABLE. Instead, we shall study more carefully the motion of the sign in these last transitions.

Suppose we narrate the substitution above in the following way. First, we substitute z_p for z_q in the first expression, $S(z_p, z_q)$, and then, as a result, following the directive of syntactic rules of substitution, substitute z_p for z_q in the second expression, $S(z_q, z_q)$, as well. Awkwardly enough, z_p, which first took upon itself the place and meaning of a numeral, subsequently was severed into two positions inside the function S. In this narrative, this z_p substituted into the left hand position in $S(z_q, z_q)$, does it retain the meaning of numeral, because it is obtained by substituting something that 'first took upon itself the place and meaning of a number', or does it now mean formula, because it stands in the syntactic position of a numeral

representing a formula? This question is undecidable, in the sense that there is no point in trying to choose between the two answers. The answer is that we have here **dangerous shifts of meaning**. If this narrative of the substitution is valid, then syntactic systems that permit substitution are demonstrated to be extremely likely to be imbued with **dangerous shifts of meaning**.

Things do not improve if we re-narrate the substitution procedure as the substitution of z_p for all three z_q positions at once. A single z_p, which is placeless (and if meaning is attached to syntactic position, also necessarily meaningless), turns out to be endowed with two distinct meanings, in two cases a numeral, and in the third a formula. If, indeed, nothing has meaning until it is placed (attached to a position in the text) then before the meaning is shifted by being substituted into various meaning articulating positions, it is not there at all. It is dangerously shifted, but with respect to no present precedent. It's presence is always already shifted. This narrative demonstrates Peirce's principle of indefatigability: ***Omne symbolum de symbolo***.

Either way, z_p does not simply mean a formula or a numeral. It carries within it the potential for penetrating the function S, so as to be integrated into a chain of events and formations, which relates signs to signs as formulas and numerals. z_p means through its own becoming unmotivated in materially and discursively constrained forms of verisimulation. z_p means through its force of splitting into (among other things) a formula and a numeral, itself and another, where it is not quite clear which is itself, and which is the other.

But the semiotic effect we have here is not exhausted by mere undecidability between numerals and formulas. z_p is disseminated into an unbounded proliferation. Indeed, as $S(z_p, z_p)$ does not include any free occurrences of w, it can be easily verified that

$$S(z_p, z_p) = S(S(z_p, z_p), z_p) = S(z_p, S(z_p, z_p)) = S(S(z_p, z_p), S(z_p, z_p)),$$

and that each of these z_p's can again be replaced by $S(z_p, z_p)$. The sign divides into itself and an excess, driving itself further and further inside, multiplying itself unboundedly.

Similarly, if by $\mathcal{N}$ we designate the function, which converts a formula

into the numeral that represents it, we obtain the following metamathematical equation:

$$S(z_p, z_p) = z_{\mathcal{N}(\Pi v[\neg B(v, S(z_p, z_p))])}.$$

Here again the right hand side can be substituted into the $S(z_p, z_p)$ in the right hand side:

$$S(z_p, z_p) = z_{\mathcal{N}(\Pi v[\neg B(v, z_{\mathcal{N}(\Pi v[\neg B(v, S(z_p, z_p))])})])},$$

making the formula longer and longer, driving the sign p away from itself indefinitely along reiterated substitutions, obscuring its provenance and its meaning as **leading principle**, deferring away.

The sign z_p splits, and its articulation as meaning formula or numeral loses its validity, grasp and explanatory force. Its articulation into numeral/formula roles would not help us follow the above manipulations. The meaning that the sign occupies in these formal manipulations (if it occupies any meaning at all) is an intermediate syntactic-operational meaning, no longer concerned with reference. In repetition z_p disseminates its own articulation, retroactively generating an effect that's captured by formalist readings. But it leaves what Derrida calls **trace**. It pivots around the comma, which acts like two mirrors facing each other, multiplying the space inscribed between them (around it) indefinitely.[17]

The syntactic attempt to anchor meaning to position sought to protect meaning against **dangerous shifts**. The assumption was that nothing was more stable than place itself; obviously, you can never move a place away from itself. But the error in this line of thinking is in imagining place as pinned down to some sort of stable ambient ether. In fact, the place is always relative to a perspective, to an articulation of boundary or the lack thereof, and to other places. The place moves with respect to the beholder. Cutting a piece of text (for the purpose, say, of substituting it somewhere) rearticulates the places carried along by the cut chunk of text with respect to the new boundaries created by the quotation marks, parentheses, or commas that inscribe it. In a textual practice, which allows quotation and

[17]It may appear that while I argue for an indefinite proliferation of the text $S(z_p, z_p)$, I accept that it has a proper starting point, the initial text $S(z_p, z_p)$. I defer to the final section of this chapter my denial of this stance.

iteration — that is in any textual practice — place is just as mobile as the sign that occupies it.[18]

The semiotic processes that take place in mathematical texts are obviously not identical to those that take place in other texts. In contemporary mathematics the primary authoritative warden is syntax. In other discourses meaning may be warded by other formative agents. To appreciate the relation between, on the one hand, the semiotic effects of mathematical repetition and self substitution, and, on the other hand, some semiotic effects in other areas of language (without claiming that one is reducible to the others!), I would like to study an example that may, at first glance, appear completely unrelated.[19]

This non-mathematical case of apparent repetition and self-predication is Jay Livingston's and Ray Evans' popular song **Que sera sera**. The chorus reads: **Que sera sera, whatever will be will be, the future's not ours to see, que sera sera**. The poetic structure and context (which I omit) indicate that we have here a double translation. First, **Que sera** is translated as **whatever will be** and the repeated **sera** as **will be**. Then **que sera** and **whatever will be** are transcribed as **the future**.

[18]This inherent instability of place does not depend on 20$^{\text{th}}$ century relativistic physics or on Leibniz' metaphysical adventures. Eriugena, a 9$^{\text{th}}$ century philosopher, claims that **the definition [of bodies and of things devoid of reason] are nowhere but in the rational soul. In it therefore will also be the places of all things that are comprehended in place. But if the rational soul is incorporeal, which no wise man doubts, it is plain that whatever is understood in it must be incorporeal; [and] place is understood in the soul, as has already been determined: therefore it is incorporeal** (Eriugena 1978, 135). Eriugena displaces place from the world into the rational soul in order to protect it from the motion it would have to suffer were it in the world. On this side of the linguistic turn, however, such naïve form of ideality is foreclosed. On this side of the linguistic turn, ideal place would be bound to the sign that occupies it, and drift along with it.

[19]Before we do this we should raise the question of generality. Is my analysis confined to these specific texts by Gödel, or is it generally applicable to mathematics? I personally am uncomfortable with analyses that are not concrete. I do believe that much of my analysis is generalisable, but concede that any specific claim above might depend on the singularity of the texts I analyse. The philosophical challenge is to relate this singularity to the singularities of other mathematical texts, and experience the plurality of mathematics in the process. No guarantee can be given a priori for the success of such a venture. This is partly why I believe that generalising my claims is no less an ethical than a rational choice.

So far everything is rather dictionary-like. But the further transcription of the next **sera** and **will be** as **not ours to see** is beyond any dictionary. When everything is recomposed back to the original **que sera sera**, some **dangerous shifts of meaning** are deeply wedged into the repetition.

But this shift of meaning does not depend merely on poetic licence. I believe that even in a casual conversation with speakers, who don't have a strong grasp of the idiomatic aspects of Spanish or English, the apparently gratuitous repetition in **whatever will be will be** will typically be understood as first referring to what is about to come, and then as expressing an inability to see into the future.

So, what does **sera** have to do with restricted vision? Let's try to disperse some of the mystery around the semiotic process folded into this little verse. We can look at this statement through Grice's concept of *implicature* or through Sperber and Wilson's concept of *relevance*. When I say that **whatever will be will be** I violate the maxim of quantity: I do not give any additional information. I also fail to provide a relevant answer to the question that appears in the song: **what will I be?** This violation is rectified if I infer that the information inscribed in the apparently irrelevant and uninformative **que sera sera** is simply this: that I don't know what you will be. This is how the meaning of **will be**, the future, is linked to ignorance and to the lack of prescience.

And yet, the additional meaning of **sera** as lack of prescience, while inferable from a well articulated theory, is not a simple product of the sign **sera** and its syntactic role. It is a product of a semiotic quest for reference, of the failure of this quest, and of the substitution of this failure for the object of the quest (in a sort of Wonderland il/logic, if one finds nothing, then nothing is what one was looking for). For subsequent use note that this post-factum theoretical explanation of the transcription of **whatever will be will be** as **the future's not ours to see** is actually not required knowledge for someone to use that phrase in that sense. The theoretical explanation hovers over the fact of use.

Since **Que sera sera** may appear to be an irrelevant example here, I will insert a missing link between the mathematical text and the poetic one. This missing link is the well-known paradox of the liar, or of the statement that states of itself: **This statement is false**. In the poetic text it

was asserted of **whatever will be** that it **will be**. In the paradox a statement imposes itself on itself.[20] The relation between the paradox and our mathematical example **leaps to the eye**, as is professed by Gödel himself (1931, 149). In all three examples — the liar, the song and Gödel's proof — when something says of itself nothing more than itself, an effect arises of lack of knowledge. Russell and Whitehead's exclusion of self-reference from mathematics is one more example of this effect. Kripke's theory of truth, which relates self-reference to a truth value that may be interpreted as undecidability, is yet another.

Returning to the mathematical text, Gödel shows that from the provability of $S(z_p, z_q)$ one can derive that $S(z_q, z_q)$ is unprovable. From this one can go on to derive from the provability of $S(z_p, z_p)$ that $S(z_p, z_p)$ is unprovable. But this derivation is every bit as a-posteriori, hovering over the fact, as is the above Gricean or Relevance Theoretic analysis of semiosis in **que sera sera**. This derivation is, of course, necessary if we are to operate a deductive system, as do Gödel, Grice and Sperber & Wilson. But it is not quite necessary for our use of the text. The self substitution of the numeral p, representing a formula, into that very same formula, as self-substitution, as self-predication, is already as linked to ignorance as the self predication of **will be** or of **this statement is false**. The link between self-substitution, self-predication and other forms of gratuitous repetition on the one hand, and lack of knowledge on the other, does not require a mathematical proof. It precedes, conditions and inspires Gödel's construction of his undecidable formula. Undecidability here is not only an effect of Gödel's proof, but also part of a more general semiotic effect of gratuitous repetition. *It is the superposition of both effects that makes Gödel's proof function as an intelligible mathematical text.*

But this picture of repetition as manifesting lack of knowledge is only half the story. Controlled repetition is in fact so intensely productive that

[20]This characterisation might appear too strong. One may assert that all we do is impose a predicate (false) on a given entity (the statement), in the same way that we may impose a predicate (false) on the statement 'the earth is flat'. However, the statement does not exist except as manifestation of falsehood. It cannot be stated independently of its being false. There is nothing to the statement except that it is false. In this sense, what it predicates on itself is itself. One may say that it manifests the predicate, rather than attributes it.

it can produce effects of intention. **Even at the babbling stage the phoneme group /pa/ can be heard**, explains Lévi-Strauss. **But the difference between /pa/ and /papa/ does not reside simply in the reduplication: /pa/ is a noise, /papa/ is a word. The reduplication indicates intention on the part of the speaker; it endows the second syllable with a function different from that which would have been performed by the first separately, or in the form of a potentially limitless series of identical sounds /papapapapa/ produced by mere babbling. Therefore the second /pa/ is not a repetition of the first, nor has it the same signification. It is a sign that, like itself, the first /pa/ too was a sign, and that as a pair they fall into the category of signifiers, not of things signified**. (Lévi-Strauss 1969, 339–340).[21]

Such an effect, the manifestation of a privileged site of meaning, appears in the other examples that we reviewed above as well. Doris Day's **Que sera sera** was originally performed for Hitchcock's *The Man who Knew Too Much*, and, indeed, the statement that **the future's not ours too see**, on top of its effect of ignorance, potentially indicates privileged knowledge. If **the future's not ours to see**, then the grammatic possibility of **the future** being someone else's **to see** emerges even before we are required to articulate this other privileged site/sight as God, leader, fate, chance...

A similar effect arises in some 'solutions' of the liar's paradox (e.g. Lacan 1978, 189), which attempt to sever the subject of enunciation from the enunciated subject (the statement that asserts falsity from the statement of which falsity is asserted). Here again a privileged site emerges, that of a subject dominating the statement-object.

This effect becomes manifest in Gödel's text, when Gödel decides, openly violating the formal syntactic framework, to derive from the fact that $S(z_p, z_p)$ is unprovable, and from the fact that it states that it is unprovable, that it is true (1931, 151). Gödel transgresses the confines of ignorance, and claims deliverance to truth beyond the confines of syntax. An extra-syntactic domain of privileged access to truth is formed. But this

[21]Several parents have protested that this statement is false. I include it anyway, because I believe it is more than simply an empirical observation.

violation of syntactic authority and emergence of a higher truth, this very emergence too, we must recall, is also subject to some syntactic system — the logic of 'natural' or 'metamathematical' language. And just as in the Gricean and Relevance Theoretic analyses of **Que sera sera**, as well as in the above 'solution' of the liar paradox, a theoretical syntactic or semantic regulatory framework may be re-constructed to contain the transgressive rapture to truth beyond syntax.[22]

That these two effects, ignorance and transgressive rapture, are intimately related is demonstrated in Derrida's reading of Kierkegaard's ***Tout autre est tout autre*** (literally, Every other is every other). This statement relates total ignorance of the other to a paradoxical source of responsibility: if my decisions do not depend on knowledge or on the authority of an other, then these decisions are mine; I am responsible for my ignorant decisions. I bring this text here to emphasise the ethical moment of the semiotic effects under investigation. **Is not this dictum — *tout autre est tout autre* — in the first place a tautology? It doesn't signify anything that one doesn't already know** unless one brings **to bear upon it an interpretation that would distinguish between the two homonyms *tout* and *tout*, an indefinite pronominal adjective (some, someone, some other one) and an adverb of quantity (totally, absolutely, radically, infinitely other)**, resulting in the interpretation: 'every other one is completely other'. **But ... then one must also distinguish between the two *autres*. If the first *tout* is an indefinite pronominal adjective, then the first *autre* becomes a noun and the second, in all probability, an adjective or attribute. One no longer has a case of tautology, but instead a radical heterology; indeed this introduces the principle of the most irreducible heterology. Or else, as a further alternative, one might consider that in both cases (tautology or heterology, with or without the homonym) the two *autres* are repeated in the monotony of a tautology that wins out after all, the monotony of a principle of identity that, thanks to the copula and sense of being, would here take over alterity itself, nothing less than that, in order to say: the other is**

[22] Such reconstruction was indeed carried through by Tarski and by Carnap, and cited in section 7 of Gödel's 1934 text.

the other, that is always so, the alterity of the other is the alterity of the other. And the secret of this formula would close upon a hetero-tautological speculation that always risks meaning nothing. But we know from experience that the speculative always requires a hetero-tautological position. That is its definition according to Hegel's speculative idealism ... The hetero-tautological position introduces the law of speculation, and of speculation on every secret (Derrida 1995, 82–83).[23]

The effect of repetition in $S(z_p, z_p)$ is every bit as dramatic as the transition from 'statement number so-and-so is unprovable' to 'I am an unprovable statement', whence self-reference, which effects both ignorance and privileged meaning, and then appeals to transgression and rapture, emerges where there was never as much as a self, only an enumerated list of formulas. The miracle of self consciousness (**Wo Es war, soll Ich werden**) is evoked here, and no amount of astute formalism commanding to turn a blind eye can take away from Gödel's proposition its self-reference as manifested when Gödel's argument is presented and explained. This self-reference is indeed avowed by Gödel himself in (1931, 149) and (1934, 362), only to be immediately disavowed (1931, 151, ff. 15).

$S(z_p, z_p)$ manifests privileged meaning just as **/papa/** manifests an intention lacking from **/pa/** and as **Que sera sera** and the liar derive privileged sites knowledge by transgressing ignorance. $S(z_p, z_p)$ turns old, habitual patterns and noises (such as $S(z_p, w)$, 'this not-yet-specified state-

[23] Another instance of relating vacuous repetition to privileged meaning is Maimonides' interpretation of the repetition in the statement **I am that I am** as a manifest proof of the necessary existence of God: **The first noun which is to be described is *ehyeh*; the second, by which the first is described, is likewise *ehyeh*, the identical word, as if to show that the object which is to be described and the attribute by which it is described are in this case necessarily identical. This is, therefore, the expression of the idea that God exists, but not in the ordinary sense of the term; or, in other words, He is "the existing Being which is the existing Being", that is to say, the Being whose existence is absolute. The proof which he was to give consisted in demonstrating that there is a Being of absolute existence, that has never been and never will be without existence** (Maimonides 1904, I.63). Note that my conflation of logic and theology is not entirely out of context. Gödel himself worked on formalising Anselm's ontological argument (Gödel 1986–2003, Vol. III, 403–404).

ment is unprovable') into a statement that explodes with novelty, not because $S(z_p, z_p)$ or 'I am an unprovable statement' is any more expressive than 'this not-yet-specified statement is unprovable', but because it links to a chain reaction of a sign with itself in different positions, which traverses language from poetry and ethics, through everyday use, to the infant's first words, encompassing some of the most challenging logical paradoxes, forcing mathematical logic to accept what until a few years earlier had never even been formulated inside it as a question: that there is an undecidable statement (and, according to Gödel, that this undecidable statement is true).

A portrayal of mathematical meaning and truth as bound to everyday semiotic processes (such as enumeration's **power of enchantment** as well as its constitution of knowledge, and repetition's manifestation of both ignorance and privileged meaning); a portrayal of mathematical discourse as necessarily imbued with **dangerous shifts of meaning**; a portrayal where mathematical meaning and truth are, on the one hand, subject to syntax, but, on the other, can exceed any given syntactic determination; a portrayal where even this excess of mathematical truth and meaning with respect to the syntax that is supposed to contain them, even this excess can be a-posteriori contained by syntactic and semiotic constructions — such a portrayal appears to me much more decent and valid than the images of clarity and distinctness that usually appear in the literature. This is all the more so if such a portrayal draws attention to the ethical impact of the motility of the sign (as I shall further attempt below), and depicts all of the above as inevitable and productive surges of constrained and disseminated responsibility, rather than as hindrances to be circumvented by some longed for, self-fulfilling, authoritative *Characteristica Universalis.*

2.8 Variables

In Gödel's text the variable is the unstable ground upon which the above operations are inscribed. It is there to be replaced. If I treated the text $S(z_p, w)$ above as a numeral, it was by abuse of terminology. In order for it to become a numeral w must be substituted. It is a **screen: at once the visible projection surface for images, and that which prevents one**

from seeing the other side. It is a place holder. It says: substitute here. **Holds in reserve and exposes to view** (Derrida 1993, 314). And the place it covers over is surrounded by other signs, which push and pull the variable screen so as to reform the image projected upon it. Indeed, as we saw above, at times the variable becomes a numeral, at other times a formula; it sustains both, as well as the projected 'mitosis' and dissemination of these terms by further self substitution.

The variable does not discriminate. Anything that fits its frame (a syntactic prescription) is welcome. It does not reject substitutions that yield falsities. **Insofar as it ingests, absorbs and interiorises everything, proper or not**, the variable, like a **sponge, is certainly "ignoble". Like its name, it takes in water everywhere**. But it is far from neutral; **it can also, when applied to a surface, expunge, wipe and efface** (Derrida 1984, 72).

This variable as surface and screen, or as tain of this mirror **thus reflects — imperfectly — what comes to it — imperfectly** — It is **the ghost of what it reflects, the shadow deformed and reformed according to the figure of what is called present: the upright fixity of what stands before me; "*the inscriptions... appear inverted, righted, fixed*.** The variable is destined to become a constant (or to be bound, but I will not dwell on this interesting and complex operation here). The operation of substitution is **what lifts the veil screen** (Derrida 1993, 314) to produce not alētheia, but the hall of funny mirrors where alētheia is inscribed. As if **mirrors would not be *in* the world, simply, included in the totality of all *onta*, and their images, but that things "present", on the contrary, would be in *them*** (Derrida 1993, 324). The entire blank page is a variable that may be substituted by a text. The place (in the world or in the rational soul) is a variable that may be substituted by a blank page.

The variable mimes. It mimes the operation of the constant that is to replace it. But the miming is so perfect, that the variable and the constant sometimes cannot be distinguished. Where Gödel writes **by** $[\alpha; n]$ **we denote the formula that results from the class sign** α **when the free variable is replaced by the sign denoting the natural number**

n^{24} (1931, 149), n is a variable — it can stand for any number. On the other hand, n is said to be a **natural number replac**ing a **free variable**, not a variable, and a numeral sign in the formal language can denote **the natural number** n only if n is a specific constant. It seems that if Gödel were to be a little more careful, he should have written **the free variable is replaced by the sign denoting the natural number** substituted for n, leaving the actual natural number in abeyance.

But in fact it simply doesn't matter. There is no formal syntactic rule to distinguish a free variable from a constant (no equivalent of the syntactic rule that distinguishes free and bound variables), apart from an explicit meta-declaration, which is often not provided explicitly. The free variable and the constant cannot be distinguished from 'inside the text'.[25] Constants and free variables can only be *told* apart. And, indeed, nothing changes in the formal appearance of things when n is substituted on the same page by **a certain** specific **natural number** q, except for the designating letter. And since this introductory part of the text does not yet fix the specific details of a formal system, q still is, to an extent, a variable depending on the precise formal system to be used.[26] **That is how the mime operates, whose act is confined to a perceptual allusion without breaking the ice or the mirror: he thus sets up a medium, a pure medium, of fiction** (Derrida quoting Mallarmé in Derrida 1993, 294).

The variable screen on which the constant projects has, as we saw in the previous section, the capacity to substitute something that contains itself into itself (the substitution of $S(z_p, z_p)$ for z_p in $S(z_p, z_p)$). It can self-divide, replicate itself, refer to itself. This mime of substitution **mimes reference. He is not an imitator; he mimes imitation** ... **In this perpetual allusion** ... **one can never know what the allusion alludes to, unless it is to itself in the process of alluding, weaving its hymen and manufacturing its text. Wherein allusion becomes a game conforming only to its own formal rules** (Derrida 1993, 219).

[24] For us here **class sign** is formula, the **free variable** is w, and **the sign denoting the natural number** n is the numeral denoted by z_n.

[25] Actually, even a variable, which appears free within a specific chunk of text, can be bound in the context of a larger chunk of text.

[26] Is an incontestable constant, such as 0, really a constant? Not to the extent that it can stand for the minimal element of any model of the natural numbers.

All this happens on the hymen surface of the variable, around the comma, between parentheses. Does it matter that the comma, the focal point of self replication is in fact absent from the formal system? Does it matter that a proper formal text in the system that Gödel employs contains no commas, that commas are confined to the shorthand that covers over the formal text? Does it matter that Quine (1995, chapter XXIII) has devised a formal system of predicate calculus with no commas or variables? But just as ladders can be thrown away, so the screen, the mirror, its tain, the hymen are all dispensable. The **irreplaceable character of** the variable, **which everything seemed to grant to it, was laid out like a trap** ... **it produces its effect first and foremost through its syntax, which disposes the "*entre*"** (between) **in such a way that the suspense is due only to the placement** ... **It is the "between", whether it names fusion or separation, that thus carries all the force of the operation** (Derrida 1993, 220).

It is not the variable, it is the gap. What underlies all these effects is the gap that allows to rupture the text, sever a syntagma, and graft it somewhere else, regardless of what it once represented, what it represents now, what it will go on to represent (formulas, numerals, self-reference, undecidability...). **This force of rupture is tied to the spacing [*espacement*] that constitutes the written sign: spacing which separates it from other elements of the internal contextual chain (the always open possibility of its disengagement and graft), but also from all forms of present reference (whether past or future in the modified form of the present that is past or to come), objective or subjective. This spacing is not the simple negativity of a lacuna but rather the emergence of the mark** (Derrida 1988a, 9–10). **One no longer even has the authority to say that "between" is a purely syntactic function. Through the re-marking of its semantic void, it in fact begins to signify** ... **Its semantic void *signifies*, but it signifies spacing and articulation; it has as its meaning the possibility of syntax** (indeed, it is the operation with semantically meaningless terms that highlights most the rules of pure syntax, those that require cutting along the spaces between the signs in order to substitute and apply syntactic rules). And by ordering the application of verisimulating syntax this gap

orders the play of meaning. *Neither purely syntactic nor purely semantic*, but rather the point where syntactic verisimulation, which is a necessary condition for effecting meaning, is found to depend on the semantics of understanding and applying rules, **it marks the articulated opening of that opposition** (Derrida 1993, 222).

It is the sustained, discrete violence of an incision that is not apparent in the thickness of the text, a calculated insemination of the proliferating allogene through which the two texts, the grafted and the grafted-onto, **are transformed, deform each other, contaminate each other's content, tend at times to reject each other, or pass elliptically one into the other and become regenerated in the repetition, along the edges of an *overcast seam* {*un surjet*}. Each grafted text continues to radiate back toward the site of its removal, transforming that, too, as it affects the new territory** (Derrida 1993, 355). The fact that grafting and substitution are syntactically defined does not prevent them from rearticulating the meaning of both the graft and the host. And the proof of that is in our reading of $S(z_p, z_p)$.

2.9 Across the sign of equality

In many ways some of my work here would have been easier, had I focused on equality rather than the variable. The motion across equality, as in

$$S(z_p, z_p) = S(S(z_p, z_p), S(z_p, z_p)),$$

is modelled after and mimes the copula. Equality operates by erasing a difference, by allowing both terms that surround the equality to communicate and become each other. Equality is not there in advance, because, a-priori, the texts around the equality are recognised as different; these texts would, for instance, be assigned different numbers in a Gödel-like enumeration of formulas. But since equality is not there (not established) until it is traversed (until the texts surrounding it are shown to be the same), one may conclude that **it is never traversed at all** (Derrida 1993, 353). It is not there, established, until it is traversed, and therefore at the moment of its

appearance as established it must already be traversed. Whenever it holds, it is already traversed. It is always already traversed.

At some point, Wittgenstein was fretful over the instability of equality.[27] **Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs** (Wittgenstein 1922, §5.53). **The identity sign is therefore not an essential constituent of logical notation** (Wittgenstein 1922, §5.533). **And we see that the apparent propositions like: "$a = a$", "$a = b.b = c. \supset a = c$", "$(x).x = x$". "$(\exists x).x = a$", etc. cannot be written in a correct logical notation at all** (Wittgenstein 1922, §5.534). But contemporary mathematics did not seem to be moved.[28] Perhaps it was not so moved, because contemporary mathematics accepted (latently, unconsciously, and against the most severe forms of astute logic represented by young Wittgenstein) that **This being-traversed is not something that happens by accident to the mirror** of equality; **it is inscribed within its structure. This is as much as to say that, forever producing itself, it never comes to be. Like the horizon** (Derrida 1993, 353).

The difference between a traditional conception of mathematical semiosis and the one I am attempting to establish here can, perhaps, be made to **rest upon the copula** (Derrida 1993, 353). The form of equality **essentialises the text, substantialises it, immobilises it. Its motion is thus reduced to a series of stances and its writing to a thematic exercise**. But to go beyond this traditional stance **It is not enough to install plurivocity in order to recover the interminable motion of writing. Writing does not simply weave several threads into a**

[27] I am abducting Wittgenstein's text here twice. First by neglecting its relation to the work of Frege and Russell, and second by ignoring the possible differences between identity and equality. Both aspects are relevant for this analysis, but I leave them out in order not to digress too far.

[28] Had we listened to Wittgenstein, who set rules to prevent **dangerous shifts of meaning** (e.g.: **write ... not "$f(a, b)$. $a = b$", but "$f(a, a)$"** (Wittgenstein 1922, §5.531)), rules that deny different variables the right to host identical values, could we then have avoided the monstrous transformation of $S(z_p, z_q)$ into $S(z_p, z_p)$? Perhaps. But I doubt that we could practice mathematics with these rules; and if we could, then monsters would most likely emerge from the friction between the sign and the gaps that would articulate it.

single term in such a way that one might end up unravelling all the "contents" just by pulling a few strings (Derrida 1993, 350).

The point of dissipation can be recognised in the following question (but doesn't have to be, for it is nowhere among the *onta*): does the copula interrelate pre-existing entities eventually discovered to have already resembled each other, or does it install **resemblance beyond difference**? There are at least two ways to answer this question:

1. The copula interrelates pre-existing entities eventually discovered to have already resembled each other (this is polysemy: an articulated closed range of meanings that can belong to a certain sign).

2. I cannot answer, because the question keeps undeciding itself. I cannot answer, because whenever I pretend to install resemblances and differences, they escape (this is dissemination: the range of meanings is not confined, it rearticulates the very attempts to exhaust and articulate it).

The difference between discursive polysemy and textual dissemination, between the two answers above, **is precisely *difference* itself, "an implacable difference". This difference is of course indispensable to the production of meaning (and that is why between polysemy and dissemination the difference is very slight). But to the extent that meaning presents itself, gathers itself together, says itself, and is able to stand there, it erases difference and casts it aside** (Derrida 1993, 351).

Wittgenstein would be right to comment that the above question is meaningless, because it exceeds the rules of the logical language game. But for those of us, who refuse to give up a question just for the mere trifling fact of its meaninglessness, this is a place where meaning can be made to spring up: in refusing the *authority* of pre-existing entities. *This is an ethical moment.*

2.10 Gödel's undecidable formula doesn't exist

And modern mathematics embraces such refusal almost to the point of dissipation. Even if we could decide whether the undecidable formula was a formal text, a number, the numeral representing it, or any former or further encoding,[29] there is still no undecidable proposition. One might have been led to believe that there was, had Gödel not devoted no less than five different footnotes to stress the claim that, despite the fact that the undecidable proposition is only denoted, represented, abbreviated in the text — despite all this the undecidable proposition can be written down. But it can't. It's too long. It contains too many signs to be written down. And so does $S(z_p, z_p)$. And so does z_p alone. All we can do is denote them, represent them, and abbreviate them.

I will not resort here to the party-trick of comparing the number of signs in the formula to the number of particles in the universe, because I don't believe that particles in the universe are numbered, and because even if the former number were smaller, it would still be too large for the undecidable proposition to be written and read by humans. And no, it does not matter either, whether the formula can or cannot be produced by a physically viable machine (one which does not violate thermodynamic laws), and which would be reliable enough to write the undecidable formula correctly with high probability.[30] It doesn't even matter whether Gödel's argument can be modified to provide a much shorter undecidable formula (indeed, it can). The only thing of importance here is that contemporary mathematics endorses Gödel's construction as it stands in the 1931 and 1934 texts regardless of whether it can or cannot be actually written down in the formal system *PM*. That is what mathematical discourse does today: it writes over.

But the issue at hand isn't just a technical strategic choice of mathematical discourse. No extreme form of finitism could resolve the difficulty above. For even if we were able to produce the undecidable formula, we

[29] See subsection 2.3 of the first chapter of this book.

[30] This kind of line of thinking can be found, for instance, in chapter 3 of Rotman (2000).

would still have to somehow verify, or consume, the construction. And it doesn't matter either whether our ability to verify the construction would or would not depend on mechanical means. None of this matters because the construction is always too complex to be digested all at once, in one moment of present epiphany. The reader (verifier, consumer) of the formula will always be broken between a self and the tools (pen and paper) used in verifying or reading the formula. And even if we were to learn the formula by heart, so as no tools may stand between the formula and our selves, we would still be split between a past where we began to chant the formula, and the future where we will end this chant. We change so much between the beginning and the end.[31] Recall the moment when Gödel announces that **the meaning of the symbols is immaterial, and it is desirable that it be forgotten**. We have already noted that willing to forget is not a likely strategy. At this point, the objective is not to fulfil desire, but to let desire operate as a mechanism of generating the complex temporality that allows joining and disjoining text and meaning. Such temporality undermines the *presence* of meaning in a text. It places meaning **between desire and fulfilment, perpetration and remembrance: here anticipating, there recalling, in the future, in the past, *under the false auspices of a present.* That is how the mime operates, whose act is confined to perpetual allusion without breaking the ice or the mirror; he thus sets up a medium, a pure medium, of fiction** (Derrida quoting Mallarmé in Derrida 1993, 294). And this splitting of the reader between a future and a past would hold, even if the formula consisted of a single digit integer. For even a single digit integer is always linked to its non-present past and future: the time when we learnt to recognise the sign, the meaning it once had, the meaning it could have, the time when we will use it next.

Among the two paragraphs above, the first is within the reach of contemporary mathematical discourse (empirically or scientifically grounded finitist critiques, such as Rotman's cited above), whereas the second is framed within post-structural philosophy, which, on a sociological level, is quite foreign to it. With these two paragraphs I am about to conclude this

[31] For a serious presentation of this provocation I refer the reader to the discussion of the effect of time on Husserl's concept of presence in Derrida's *Speech and Phenomena* (Derrida 1979).

chapter (I only have 13 marked quotation left, which I'd like to include. I may give some of them up, because I am tired, and so, I believe, are you). In reading what follows, **it is desirable** (I so desire) that you perform a double reading. Once forgetting the second (post-structural) paragraph, and once remembering it; once recalling that the undecidable formula does not exist because it is simply too large, and once recalling that it does not exist because it keeps disseminating between its possible histories and future references. The first approach would reflect material constraints that curtail unbounded recursions and the authority of non-presentable formal texts, as well as such issues of underdetermination as the Löwenhim-Skolem 'paradox' and the openness of increasing and bifurcating logical hierarchies. The second approach would relate mathematics to post-structural concerns with the unbounded dissemination of marks, decisions and closure. The choice of reading that you may (or may not) make is ethical. After all, **he who understands me finally recognises** that semiotics is **senseless**. And despite the fact that **There can be no ethical propositions** (Wittgenstein 1922, §§6.54, 6.42), the proposition that I make in this essay, I make as entirely ethical. It is ethical in its concern with the privileged authority of mathematics as discourse, some of it well earned, but some based at least in part on the myth of its unified, well-grounded semiotic stability.

The 1931 and 1934 texts **don't represent anything that has ever been or can ever become present: nothing that comes before or after the mimodrama**. The mimodrama is that of a formal text, **which has never been committed**, has never been properly written down as such, **and yet nevertheless turns into a suicide** of Hilbert's programme to establish consistency through finitary means. The fully elaborated formal text was never written, yet still it has authority over that realm of unwritten formal texts, an authority that overturns Hilbert's aspirations, without an epistemically violent revolution of the discourse, **without striking or suffering a blow, etc.** (Derrida 1993, 210). The structure of the comma, variable, equality and gap of substitution (the hymen, the *entre*) that enables mathematical writing lives in texts, 'on paper', indicating something that can't be brought forth. It only takes **place when it doesn't take place, when nothing *really* happens, when there is an all-consuming consummation without violence, or a violence without blows, or**

a blow without marks, a mark without a mark (a margin), etc., when the veil, *without being*, torn, and when the prosecution has nothing except a written proof, a written confession (social scientists know well that if you torture the numbers, they will confess).

We have already seen that some particles of the text (such as variables and equality signs) act like mirrors — but mirrors that do **indeed come to stand as a source, like an echo that would somehow precede the origin which it seems to answer — the "real", the "originary", the "true", the "present", being constituted only on the rebound from the duplication in which alone they can arise** (Derrida 1993, 323).[32] **This double mark escapes the pertinence or authority of the truth: it does not overturn it but rather inscribes it within its play as one of its functions or parts**, just as Carnap, Tarski and Gödel inscribe truth in their open hierarchy of meta-languages, each language in the hierarchy articulating the truth of its object language — but more importantly, as each shorthand text derives its authority from a longhand transcription that needn't exist, or needn't be present-able. **This displacement does not take place, has not taken place once, as an *event*. It does not occupy a simple place. It does not take place *in* writing. This dis-location (is what) writes/is written**. It is not in the text. It is in writing about the text, in reading it, in its use, its reiteration (Derrida 1993, 193).

When, in Gödel's 1934 text, it was **desirable that** the meaning attached to symbols **be forgotten**, we found that it didn't actually matter whether we forgot or not, whether we could forget or not, whether we could fulfil this desire to forget. It was the desire that the meaning of symbols be forgotten that was enough to manipulate the interaction between meanings and texts. Therefore, **there is no longer any difference between desire** ... **and the fulfilment of presence** ... **there is no longer any difference between desire and its satisfaction**. And this is closely related to the dysfunctionality of the difference between an actual presentation and a mere indication of Gödel's undecidable formula, which by now

[32]Kristeva quotes Lenin: **It is exact that men begin with *that* (the natural principle), but truth is not in the beginning, but in the end, more precisely in the continuation** (Kristeva 1969, 213).

should have become undecidable in a more than formal sense.

It is not only the difference (between desire and fulfilment) that is abolished, but also the difference between difference and nondifference. Nonpresence, the gaping void of desire, and presence, the fulfilment of enjoyment, amount to the same. By the same token {*du même coup*} in Gödel's proof **there is no longer any textual difference between the image and the thing, the empty signifier and the full signifier, the imitator and the imitated, etc.** We saw how z_p loses footing when we ask what it means as it goes through processes of substitution, and how it stands for something that could never be made present. **But it does not follow, by virtue of this hymen of confusion, that there is now only one term ... it does not follow that what remains is ... the imitated, or the thing itself, simply present in person**. It does not follow that all we have is the plain mark z_p, **simply present** (Derrida 1993, 209). Without its meaning and function of representation z_p alone cannot enable an intelligible proof concerning formal systems. Mathematical discourse simply does not require the difference between desire and fulfilment, between present and absent, to function in a way that would completely dominate practice. But *dys*functional difference is **indispensable to the production of meaning**, to our ability to produce effects of desire and consummation, of presence and absence of meaning and intelligible proof. All these notions are disseminated by dysfunctional differences through inscription under the auspices of mathematical discourse.

Structure (the differential) is a necessary condition for the semantic, but the semantic is not itself, in itself, structural. The seminal, on the contrary, disseminates itself without ever having *been* itself, and without coming back to itself. The process of substitution of z_p, its **very engagement in division, its involvement in its own multiplication**, its substitution into itself, self-reference, unlimited nestings of $S(z_p, z_p)$ inside itself, its denoting of a text that cannot be forced into presence, the effect of undecidability, this **is what constitutes** the mathematical mark **as such in its living proliferation. It exists in number** (Derrida 1993, 351).

What is this **number**? Number is what counts oranges, number is

what measures voltages. But oranges get squashed into uncountable mush, and voltages are subject to 'measurement errors'. Number is merely indicated by the empirical. The diametrically opposite alternative to this conception is to take Peano's approach: numbers are unbounded collections of signs that obey certain syntactic rules — syntactic rules that nothing *present* can ever properly fulfil; this solution is that which contemporary mathematical scholarship usually refers to under the title of its *foundations*. Nevertheless, this merely indicated column of numbers, the exhaustive, mechanical enumeration of all formulas in a formal system, is the centre-piece of Gödel's text. It ties together the various strata of the argument (arithmetic, formal systems, metamathematics). But this column ***has*** **no being, nor any being-there, whether here or elsewhere. It belongs to no one** ... **you will never absolutely control its extension. You will not take it from somewhere else and put it here. You will not cite it to appear. Yet despite this column's not being (a being), not falling under the power of the *is*, all of western metaphysics, which lives in the certainty of that *is*, has revolved around the column. Not without seeing it but on the contrary in the belief that it sees it. And can be sure, in truth, of the contours of its collapse, as of a centre or a proper place** (Derrida 1993, 352, translation modified).

All this does not deny numbers and mathematical marks their usefulness: they do allow us to count and measure. This should, however, make an impact on their authority. If we accept that mathematical semiosis shares so much of its unstable generative processes with other forms of language, it makes little sense to assume that it has an a-priori privilege. The authority of mathematics should therefore be judged according to its applications and results (use value, exchange value), rather than its pretensions to superior form.

It's as a web of unfounded indications that mathematics becomes unboundedly usable, exchangeable without being given up, inexhaustible. Acknowledging the disappearance of numbers as a present source, **the death of that representative voice**, the voice that makes numbers present, **that voice which is already dead, does not amount to some absolute silence that would at last make way for some mythical purity of writing, some finally isolated graphy. Rather, it gives rise to an**

authorless voice ... **that no ideal signified or thought can entirely cover in its sensible stamp without leaving something out** (Derrida 1993, 332). **Because a written sign** ... **is a mark that remains** (Derrida 1988a, 9, translation modified). And therefore **The movement of signification adds something, which results in the fact that there is always more, but this addition is a floating one because it comes to perform a vicarious function, to supplement a lack on the part of the signified** (Derrida 1978, 289).

Chapter 3

The Surface of Mathematics

We finally come to consider the interface between bodies and texts. Psychoanalytic theory and modern logic lead us to the surface of language, and to the lining that separates and relates words and the actions they order. There we make sense.

The idea is that when I give you an order, there are the words — then something else, the sense of the words — then your action ... And they go on to say that the series of cardinal numbers is known to us by a ground-intuition — that is, we know at each step what the operation of adding 1 **will give. We might as well say that we need, not an intuition at each step, but a *decision.* — Actually there is neither. You don't make a decision: you simply do a certain thing. It is a question of a certain practice** (Wittgenstein 1975, 185, 237). No intermediation, says Wittgenstein. No sense, no intuition, no decision between words and actions.

Wittgenstein and his disciples are right to define meaning by means of use, concurs Deleuze (1990, 146). Nothing comes *between* words and actions. But Deleuze goes on to articulate that **use is in the relation between representation and something extra-representative, a nonrepresented and merely expressed entity**. Use relates the sys-

tem of representation to a surface of expressed sense. And **representation, when it does not reach this point, remains only a dead letter confronting that which it represents, and stupid in its representativeness**.

Formal mathematical language can be dwindled to become such dead letter. It can degenerate into conglomerates of words and actions, without enveloping the event **at its borders**, without bringing **about this lining or hem** (Deleuze 1990, 146), this phantasmatic surface, this effect of meaning. It is the task of this chapter to place mathematical language, at a particular moment in its history, within the theoretical edifice of Deleuze's linguistic surfaces — surfaces that do not come between words and actions, but develop sense somewhere in their vicinity.

3.1 Numbers and the body

Let's open with an analysis, inspired by Deleuze's reappropriation of transcendental philosophy, of what makes numbers possible. Since we have to start from somewhere, the skin will be our somewhat arbitrarily chosen surface of incision. Not because the skin or the body that it envelops are taken to be unproblematic genuine givens, but because the body's inside and outside are privileged by the capacity of our scalpel — words — to cut along the skin. It is easy to articulate an inside and an outside of a body with words (if we were to use sharper tools, we might have cut through the body, and caused some pain).

Since there is nothing between words and actions, we shall begin our search for meaning within the body, which speaks and acts. And the body is explicitly there as we read Gödel's texts. Things **stand**, **hold**, **run** (into), are **carried** (through), **constructed**; we **say**, **write**, **see**, and there is even a **left-hand** (side). These idiomatic uses establish an echo of a link between the actions of the body and the meaning of a mathematical text. But even without these idiomatic links, Gödel's text is ordered around **the series of cardinal numbers** and syntactic **steps**, which build upon the embodied **operation of adding** 1.

Numbers order Gödel's text. Every string of signs in the formal language is numbered by this text. Formulas become numbers. More generally,

numbers are a privileged constituent in contemporary accounts of the foundation of mathematics. And the series of cardinal numbers is the result of the **operation** of adding 1. So numbers depend on the operation of the body (or, at the very least, of the embodied mind). Numbers are extracted from the body's **steps**. But one should be wary of hypothesising the body as the *origin* of numbers. The body is, we insist, the effect of our somewhat arbitrary verbal incision along the skin. We may have cut along a different surface, using different cutting instruments. And the body, while numbers pulsate inside it (the heart beats), while it keeps adding 1 (taking another step), yet the body alone cannot synthesise numbers. Numbers require a transcendental account.

Steps follow, and the heart beats. But that alone is not enough, as each step is different, and as each heartbeat is distinct. In order to contract steps and heartbeats into a form of repetition, a synthesis must occur. The first synthesis would be the *passive synthesis* — the contraction of instances to form a repetition. Passive synthesis **refers to the fusion of successive tick-tocks in a contemplative soul** ... **it constitutes our habit of living, our expectation that 'it' will continue** ... **When we say that a habit is a contraction we are speaking not of an instantaneous action which combines with another to form an element of repetition, but rather of the fusion of that repetition in the contemplating mind** (Deleuze 1994, 74).

We must not be so hasty as to ask here *who* is this **contemplative soul** or **mind**. Such question would presume that they have the form of a *someone*.[1] But we may ask what it is to contemplate. **To contemplate is to question. Is it not the peculiarity of questions to 'draw' a response?** ... **'What difference is there...?' This is the question the contemplating soul puts to repetition, and to which it draws a response from repetition** (Deleuze 1994, 78). **What difference is there** between the first and the second step? Between the second and third heartbeat? In order to count the repetitions, both similarity and difference must be acknowledged by a **contemplative soul**. Without asserting the differ-

[1]Here we can allow for contemplative souls as components of consciousness that are incommensurable with a so called thinking subject. Many contemplative souls may simultaneously partake in a given thinking subject and its others.

ence within repetition, we would not have $1, 2, 3, \ldots$, but merely $1, 1, 1, \ldots$, or perhaps even just 1 — a constant death.

This is what we have in a formal language, if we accept the most repressive formalist and finitist accounts. We repeat signs, we repeat mechanic procedures for determining whether a certain sequence of primitive signs is a **meaningful formula**, an **axiom**, a **proof**. This is all there is. **A formal mathematical system is a system of symbols together with rules for employing them** (1934, 346). But even in this account the formal system is not just the symbols, it is the symbols along with the rules, later to be specified as **mechanical**. And even if these rules order nothing but machines, they still incite a cohort of **contemplative souls**, which accompany the text, and **draw** from it an effect of repetition.

But even this conjunction of body and contemplative soul is not enough to produce numbers. Formal languages do not acknowledge numbers until they are written down. Deleuze's generalised formulation of this fact states that material repetition **comes undone even as it occurs, and can be represented only by the active synthesis which projects its elements into a space of conservation and calculation** (Deleuze 1994, 84). Without this **space of conservation** we cannot hold on to repetition.

In the specific context of mathematics, Derrida reads the necessity to add an active, inscriptive, retentive synthesis to the passive one in Husserl's *Origin of Geometry.* **By itself the speaking subject, in the strict sense of the term, is incapable of absolutely grounding the ideal Objectivity of sense. Oral communication (i.e. present, immediate and synchronic communication) among the protogeometers is not sufficient to give ideal objectivities their "continuing to be" and *"persisting factual existence,"* thanks to which they perdure "even during periods in which the inventor and his fellows are no longer awake to such an exchange or even, more universally, no longer alive." To be absolutely ideal, the object must still be freed of *every* tie with an actual present subjectivity in general. Therefore, it must perdure "even when no one has actualised it in evidence" (164 [modified]). Speech [*language oral*] has freed the object of *individual* subjectivity but leaves it bound to its begin-**

ning and to the synchrony of an exchange within the *institutive community.* The possibility of writing will assure the absolute traditionalisation of the object, its absolute ideal Objectivity — i.e., the purity of its relation to a universal transcendental subjectivity. Writing will do this by emancipating sense from its *actually present* evidence for a real subject and from its present circulation within a determined community. "The decisive function of written expression, of expression which documents, is that it makes communication possible without immediate or mediate address; it is, so to speak, communication become virtual" (164 [modified]) (Derrida 1989, 87; internal quotes refer to the same volume, which includes an English translation of Husserl's text)

Numbers have to be actively synthesised, transcribed onto the blank page or onto an otherwise restrictive and imposing space of representation in order to function as numbers, rather than disappear across the threshold of the contemplative soul's breakdown into fatigue (attrition, mechanical fault, computation error). Indeed, Gödel's argument cannot be performed without ***numerical symbols* or expressions representing numbers** (1934, 350).

But even that is not enough. Numbers are numbers not simply by virtue of passive contraction and active synthesis of representation. In order to go on, to keep on repeating, they require a much farther reaching form of repetition. They require an **Eternal return**. Eternal return **affects only the new, what is produced under the condition of default** (passive synthesis) **and by the intermediary of metamorphosis** (active synthesis). **However**, eternal return **causes neither the *condition* nor the *agent* to return: on the contrary, it repudiates these and expels them with all its centrifugal force. It constitutes the autonomy of the product, the independence of the work. It is repetition by excess which leaves intact nothing of the default or the becoming equal. It is itself the new, complete novelty** (Deleuze 1994, 90).

Let us try to explain. When exposed to the centrifuge of reiteration, which is called here eternal return, numbers are repeated by different agents on different surfaces of representation in different contexts. In this process, numbers lose the agent who counts them, the body where they pulsate, and

the page on which they are inscribed. But the result is far from *ideal*. Numbers do not become the platonic object independent of experience, which Frege and Husserl affirmed.[2] This is so because as we strip the repeated (or quoted, reiterated) texts bare of any specific agent and context, we are not left with a steady common denominator, but rather with ever changing forms and reforms of writing. These ever evolving forms open the text to unanticipated reformulation, rather than establish an ideal common form. What is repeated in the eternal return is this opening to new forms of writing, and to an interaction with new agents, new experiences, new uses and new contexts.

Numbers, the experience of numbers, the *repetitive* operation of adding 1, become the opening into **novelty** of that which is repeatedly different from itself. What eternal return sets before us (always in advance) is difference itself, the active difference that propels the sequence of numbers, and causes them to proceed rather than stay put. This iteration, or eternal return, produces as an effect the illusion of a division between contingent circumstances and ideal content, as well as what we have asserted above as constitutive conditions: the supposedly neutral **space of conservation and calculation** upon which numbers are represented, the illusion of their independent existence there, and, finally, the experience of passive synthesis, their contraction into instances of primitive repetition, the effect of a clear-cut repetition of tick by tock.

This eternal return is described, again, in a context closer to our own, namely in Derrida's reading of Husserl's *Origin of Geometry*. We compare here Greek mathematics with subsequent mathematical revolutions. First, in Greek mathematics, **On the basis of a finite apriori system, an infinite number of mathematical operations and transformations is already possible in that system, even if they are not infinitely creative. Above all, despite the closedness of the system, we are *within* mathematical infinity because we have definitely idealised and gone beyond the factual and sensible finitudes**. This is the *finite* infinity. **The infinite infinity of the modern revolution can be announced in the finite infinity of Antiquity's creation. While investigating the sense of what they created — mathematical**

[2]See their quotes in the opening of the first section of chapter 1.

aprioriness — the Greeks simply would not have investigated the sense of all the powers of infinity which were enclosed in that aprioriness ... That will be done only progressively and later on, by interconnecting revolutionary developments conforming to the profound historicity of mathematics and to a creativity which always proceeds by disclosure (Derrida 1989, 130).

The structure of this process of origination opens up a new form of *telos*. For **if such infinitisation is a new birth of geometry in its authentic primordial intention (which we notice still remained hidden to a certain extent by the closure of the previous system), we may wonder if it is still legitimate to speak of *an* origin of geometry. Does not geometry have an infinite number of births (or birth certificates) in which, each time, another birth is announced, while still being concealed? Must we not say that geometry is on the way towards its origin, instead of proceeding from it?** (Derrida 1989, 131).

This process of origination has a crucial role in Gödel's text. Its manifestation is the never quite completely determined anticipation of the next step in the argument, which is constrained by rigorous forms of repetition, but folds within itself the unexpected — that which the reader does not, and perhaps never could, discover by himself. From the repetition of well known mechanic manipulations arises the **novelty** of a conclusion, an overwhelming discovery and surprise. This is the opening that allows numbers to become formulas (sometimes PROVABLE, sometimes UNPROVABLE), and that allows the sequences of numbers and formulas to interrelate, while maintaining the potential of these sequences to keep on communicating with other levels of representation (arithmetic, formal, metamathematical) in ways that we do not yet anticipate. Without this opening of numbers to their rigorously restricted, yet unprecedented use, without this opening, which (according to Derrida's Husserl) would constitute the origin of mathematical objects, Gödel's argument simply could not come to be written.

Following Deleuze, we provided a transcendental account of three levels of repetition. First the articulation of concrete repetition by a 'contemplative soul' (which can be as simple as a mechanical procedure). Second, the articulation of repetition on a space of representation (whether a piece

of paper or objective ideality). Finally, the repeated opening of the entire framework to novel implementations of difference and repetition, novel acts of communication between rigorously established distributions of difference and repetition.

This transcendental account opens up a mystery. What is it that allows differences to communicate into eternal return? Is it **really difference which relates different to different in these intensive systems? Does the difference between differences relates difference to itself without any other intermediary? ... what is this agent, this force which ensures communication? Thunderbolts explode between different intensities, but they are preceded by an invisible, imperceptible *dark precursor*, which determines their path in advance ... Likewise, every system contains its dark precursor which ensures the communication of its peripheral series**. In its role as communicator between sequences of different elements, the dark precursor would have to be **the in-itself of difference or the 'differently different' — in other words, difference in the second degree, the self-different which relates different to different by itself. Because the path it traces is invisible and becomes visible only in reverse, to the extent that it is travelled over and covered by the phenomena it induces within the system, it has no place other than that from which it is 'missing', no identity other than that which it lacks: it is precisely the object $= x$, the one which is 'lacking in its place' as it lacks its own identity** (Deleuze 1994, 119–120).

Retracing the dark precursor, first in the generation of a 'natural' language surface, and then in the generation of the mathematical language surface that Gödel's texts unfold, is what we shall do next. But before we do that, it is important to understand the repercussions of this retracing. Since if we recover the dark precursor, we must ask whether **identity and resemblance** are **the preconditions of the functioning of this dark precursor, or are they, on the contrary, its effects? If the latter, might it necessarily project upon itself the illusion of a fictive identity, and upon the series which it relates the illusion of a retrospective resemblance? Identity and resemblance would then be no more than inevitable illusions — in other words, concepts of**

reflection which would account for our inveterate habit of thinking difference on the basis of the categories of representation ... **it is in the nature of the surface** of representation **to cancel difference, but only on the surface** (Deleuze 1994, 119, 240).

The question raised in this last quotation is whether eternal return is an effect, or, despite the fact that (according to Derrida's description above) it only manifests itself as the future of our previous limited systems of thought and calculation, eternal return as described above is actually seminal, disseminal, a constitutive opening — but an opening which is impossible to determine and fully represent, because it is always ahead of itself. We'll return to this question after we articulate a theory of language that can sustain it.

3.2 The dimensions of language

In order to characterise mathematical language and consider its specificity compared to other forms of language, we will use Deleuze's account of language in his *Logic of Sense*. First, we will review his analysis of the static dimensions of language. Then, since mathematical language is dynamic, we will briefly review his genetic account of language, and try to describe the formalist reduction and Gödel's reaction in terms of this dynamic account.

Our point of reference for this account of language will be mathematical language as it stood before the late 19^{th} century foundations crisis, at the verge of the discovery of paradoxes in Cantor's set theory. Of course, many mathematical languages were employed before this crisis, and each had its specific peculiarities. The analysis that follows cannot properly describe all these languages at once. Still, for the purposes of this introductory account I prefer to introduce a myth of a hypothetically reconstructed pre-crisis mathematical language that ranges from Greek mathematics to the 19^{th} century, rather than commit myself to some specific historic moment.

Many authors agree in recognizing three distinct relations within the proposition. We shall consider these relations below, following Deleuze (1990, 12–15). **The first is called denotation or indication: it is the relation of the proposition to an external state of affairs** ***(datum)*** ... **Logically, the denotation has as its elements and its**

criterion the true and the false. "True" signifies that a denotation is effectively filled by the state of affairs ... or that the correct image has been selected ... "False" signifies that the denotation is not filled, whether as a result of a defect in the selected images or as a result of the radical impossibility of producing an image which can be associated with the words.

Pre-crisis mathematics had a strong commitment to denotation. Mathematical descriptions often referred to states of affairs. They were not tied-down to them, perhaps they did not even depend on them, but, as a rule (with many exceptions, just like in 'natural' language), mathematical objects were usually reified in some way. The diagram, economic or geometric interpretation and physical observation went hand in hand with mathematical analysis. This is why the introduction of complex numbers was considered much more objectionable before the introduction of the complex plane model than once it had been established. This is why non-Euclidean geometry required models (the half-plane with semicircles as lines, the disc with circular arcs as lines, the pseudo-sphere), rather than just an axiomatisation, in order to be properly endorsed. This is probably the motivation behind Weierstrass', Dedekind's and Cauchy's attempts to ontologically reform infinitesimals and real numbers. In pre-crisis mathematics one could definitely ask whether a given mathematical description was true for a certain state of affairs.

A second relation of the proposition is often called "manifestation". It concerns the relation of the proposition to the person who speaks and expresses himself. Manifestation is therefore presented as a statement of desires and beliefs which correspond to the proposition. Desires and beliefs are causal inferences, not associations. Desire is the internal causality of an image with respect to the existence of the object or the corresponding state of affairs. Correlatively, belief is the anticipation of the object or state of affairs insofar as its existence must be produced by an external causality ... there are in the proposition "manifesters" like the special particles I, you, tomorrow, always, elsewhere, everywhere, etc. ... from denotation to manifestation, a displacement of logical values occurs which is represented by the Cogito: no

longer the true and the false, but veracity and illusion.

The dimension of manifestation may appear problematic. Mathematical language has hardly ever said 'I'. But this is a very narrow-sighted objection. Mathematics, while rarely saying 'I', has often said 'we'. Pre-crisis mathematical language often manifested a universal consciousness — be it veracious or illusionary — which then underlay various attempts to form a *Characteristica Universalis*, systems which were meant to serve as symbolic manifestations of the spirit of thought or nature (Leibniz and the Pythagoreans are two obvious examples). Mathematical axioms are manifestations, if not of beliefs, then of convictions. The order of implication was often reconstructed as manifesting an order of causality (what this act of reconstruction itself manifested is a different concern). It was 'because' of the mathematical analysis that certain phenomena had to happen (it was 'because' of the inverse square law that planets revolved in elliptic trajectories). I admit, however, that the question of desire in pre-crisis mathematical languages is more delicate, and would require a closer textual analysis.

We ought to reserve the term "signification" for a third dimension of the proposition. Here it is a question of the relation of the word to *universal or general* concepts, and of syntactic connections to the implications of the concepts ... Signification is defined by this order of conceptual implication where the proposition under consideration intervenes only as an element of a "demonstration", in the most general sense of the word, that is, either as premise or conclusion. Thus, "implies" and "therefore" are essentially linguistic signifiers ... The logical value of signification or demonstration thus understood is no longer the truth, as is shown by the hypothetical mode of implications, but rather the *conditions of truth*, the aggregate of conditions under which the proposition "would be" true ... the condition of truth is not opposed to the false, but to the absurd: that which is without signification or that which may be neither true nor false.

As for the dimension of signification — conceptual abstraction and the logical order of implication — this has been the most venerated dimension of mathematical language since Euclidean times, if not before. In

many ways mathematical language inspired (and distorted) the subsequent significational analysis of natural language. One can find in many thinkers an explicit wish to impose upon natural language the sound significational structure operative in mathematical language. The questions of 'conditions of truth' saturated mathematics long before this phrase was even coined.

These are the traditional dimensions of a functioning language as articulated by Deleuze. Language points to things, manifests a speaker, and establishes an inferential order. While trying to track down which of these dimensions establishes the others, Deleuze discovers that each already presupposes the others (I will not repeat his argument here). The question arises whether there is **something, *aliquid*, which merges neither with the proposition or with the terms of the proposition, nor with the object or with the state of affairs which the proposition denotes, neither with the "lived", or representation, or mental activity of the person who expresses herself in the proposition, nor with concepts or even signified essences? If there is**, then this additional dimension, **sense, or that which is expressed by the proposition, would be irreducible to individual states of affairs, particular images, personal beliefs, and universal and general concepts** (Deleuze 1990, 19).

Deleuze points out a long history of attempts to assert or deny such an additional fourth dimension of language. How such a dimension would be generated will be reviewed below. Deleuze opts to first track down this fourth dimension through an analysis of the relations between the three traditional dimensions of language (denotation, manifestation, signification), and through recording their 'paradoxes'. We will, however, defer an explication of the internal characterisation and explicit manifestation of this fourth dimension of sense until the *first finalé* of this chapter for technical and rhetorical reasons.

Since the task of demonstrating the role of such a dimension in an analysis of a fully-fledged language is independent of the task of explaining how it would come to be, our first approach to this fourth dimension will be genetic. We will review the narrative of how such a dimension comes to be in 'natural language', and attempt to locate the relevant parallel elements in our (mythical) language of pre-crisis mathematics.

3.3 A concise psychoanalytic genealogy of language

In the first section we associated counting and the production of numbers with embodied repetition. We saw that the **gesture that constituted the sign was a gesture of despatialisation: it reduces the volume** of three dimensional embodied experience **to a surface** of inscription, **the practice to a chain of sounds** (Kristeva 1969, 78). Active synthesis derived from the depths of the body a superficiality of text and script. The operative question is how the volume of the body ends up despatialised onto a surface. Or inversely, **without forgetting that the text presents a system of signs**, we seek to open up **in the interior of this system another scene: that which the screen of structure covers, and which is signification *as operation whose structure is but displaced fallout*** (Kristeva 1969, 279).

I will now briefly sample Deleuze's account (with some help from Kristeva; both accounts emanate from Melanie Klein and Jacques Lacan, but are not quite subject to them) of the derivation of language (the surface) from body (the volume). This digression, which will set mathematics aside for a while, is to be justified by the reintegration that will follow next. In order to keep things containable, the details that I shall provide here will be scanty and incomplete. This is acceptable here, because I only wish to set up the possibility of a narrative whereby what we shall call a *paradoxical element* produces a surface of sense.

Our first surface will be the skin (recall, however, that our initial incision along the surface of the skin, rather than along any other surface, is not an absolute starting point, but is contingent on our surgical instruments, namely words). The skin, however, is not quite our starting point. Before we have the surface of the skin, before there is an inside and an outside of the body, there is no difference between eating and speaking. **Chrysippus taught: "if you say something, it passes through your lips; so, if you say "chariot", a chariot passes through your lips"** (Deleuze 1990, 8). Food passes through your lips, sounds pass through your lips; in either case your body vibrates. The limits of the body and the limits of the world coincide; both are the limits of sensation. **As there is no surface,**

the inside and the outside, the container and the contained, no longer have a precise limit. Therefore **everything is body and corporeal. Everything is a mixture of bodies, and inside the body, interlocking and penetration** (Deleuze 1990, 87). Everything falls **into the anonymous pulsation wherein words are no longer anything but affections of the body — everything falls back into the primary order which grumbles** (Deleuze 1990, 125).

Discrete quantities of energy move through the body of the subject who is not yet constituted as such and, in the course of his development, they are arranged according to the various constraints imposed on this body ... in this way the drives, which are "energy" charges as well as "physical" marks, articulate what we call the *chora*:[3] **a nonexpressive totality formed by the drives and their stases in a motility that is as full of movement as it is regulated** (Kristeva 1984, 25).

The depth is clamorous: clappings, crackings, gnashings, cracklings, explosions, the shattered sounds of internal objects. But that is not all. Something, **from among all the sounds of the depth, ex-**

[3]**We borrow the term *chora* from Plato's *Timaeus* to denote an essentially mobile and extremely provisional articulation constituted by movements and their ephemeral stases. We differentiate this uncertain and indeterminate *articulation* from a *disposition* that already depends on representation, lends itself to phenomenological, spatial intuitions, and gives rise to a geometry** (Kristeva 1984, 25–26). **It does not have the characteristics of an existent, by which we mean an existent that would be receivable in the *ontologic*, that is those of an intelligible *or* sensible existent. There is *khōra* but *the khōra* does not exist. The effacement of the article should for the moment suspend the determination ... and the reference to something which is not a thing but which insists, in its so enigmatic uniqueness, lets itself be called or causes itself to be named without answering, without giving itself to be seen, conceived, determined** (Derrida 1988b, 237). **Although our theoretical description of the *chora* is itself part of the discourse of representation that offers it as evidence, the *chora*, as rupture and articulation (rhythm), precedes evidence, verisimilarity, spatiality, and temporality. Our discourse — all discourse — moves with and against the *chora* in the sense that it simultaneously depends upon and refuses it. Although the *chora* can be designated and regulated, it can never be definitely posited: as a result, one can situate the *chora* and, if necessary, lend it a topology, but one can never give it axiomatic form** (Kristeva 1984, 26).

tracts a Voice (Deleuze 1990, 193). The voice is distinguished from the clamour of partial objects by its **being found only as lost, of appearing for the first time as already there**. It **speaks and comes from on high** (Deleuze 1990, 193). The unique position of the Voice with respect to the clamour is the primitive emergence of the topography that opposes good to bad, height to depth, and which, between the two, embarks on developing what will later become the surface of the skin.[4]

Here is how this could take place. When the body produces noises, it also feels a rumble in the digestive system, the echo of the noises in the diaphragm. Food and noise are one; they both rumble in the stomach. But sometimes the noise is only heard, while the stomach stays still. The effect is double. First, this experience articulates the eardrum and separates it from the diaphragm. Second, it articulates this noise in the form of a Voice, namely as always having been there, as an independent outside.[5]

The Voice with its outside-inside or high-low topography already establishes the dimensions of language: it designates (an already-lost object), signifies (an order of preexistence and outside), and manifests (itself as withdrawn). But since all these dimensions relate to the lost, unknown, withdrawn, this is not yet a language. **And so we are left outside sense, far from it ... in a *pre-sense (pré-sens) of heights*** (Deleuze 1990, 194).

What we have below the Voice is not yet the unified surface of the skin; the skin does not yet exist. **In fact, each zone is the dynamic formation of a surface space around a singularity constituted by the orifice** (ear, mouth, anus, penis, vagina...). Each zone **is able to be prolonged in all directions up to the vicinity of another zone depending on another singularity. Our sexual body is initially a Harlequin's cloak** (Deleuze 1990, 197). In order to put the various zones

[4] A more orthodox Lacanian account would place the mirror before the voice. **Captation of the** child's unified body image in the mirror **and the drive investment in this image ... permit the constitution of objects detached from the semiotic *chora*.** An imaged ego is formed, whose positing **leads to the positing of the object, which is likewise, separate and signifiable ... The sign can be conceived as the voice that is projected from the agitated body (from the semiotic *chora*) onto the facing *imago* or onto the object, which simultaneously detach from the surrounding continuity** (Kristeva 1984, 46–47).

[5] We suppress the role of Deleuze's unifying **body without organs** in this narrative.

together, we require an equivalent of the **dark precursor** from the first section above.

In order to put things together (unify the fragmented body and relate the high and the low), according to Deleuze, the boy projects the image of the Voice (the form of an independent preeminent outside) upon his own penis. The *phallus* is thus constituted. Now that it is endowed with the self contained structure of the Voice, the penis-become-phallus will form a complete surface around it. It will be able to resist the violent chora and partial objects that upset the emerging inside/outside division. Instead of being detached and elevated, the power of the Voice will be forced here, on the skin, within hand-reach. The heights will be brought down to the surface, and the clamorous depths will be integrated under this surface and shut in an inside.

But this plan must fail. In trying to grab hold of the Voice and pin it onto the body, its very essence as withdrawn independent outside is betrayed. **This essence could not be found but only as if recovered — recovered *in* absence and *in* forgetfulness — but never given in a simple presence of the "thing" which would eliminate forgetting** (Deleuze 1990, 205–206). Brought down onto the body, the Voice loses its constitutive structure.[6]

In this narrative, the phallus has two aspects. The first aspect is the intention of organising the surface of the body, and summoning the withdrawn pre-eminence. The second aspect is the consequence of collapsing the structure of the Voice. These incompatible effects cannot take place on the same surface. This is precisely where the surface splits. The phallus of good intentions organises the surface of the skin into a complete unity. The phallus of the tragic consequences, that of castration, doubles the surface of the skin with a second withdrawn **screen, the cerebral or metaphysical surface on which the phantasm is going to develop, begin anew with a beginning which now accompanies it at each step, run to its own finality, represent pure events** (Deleuze 1990, 218). A surface of consequences, of independent and preeminent pure events is formed, a surface of sense.

[6] I suppress here the elaboration of this process as castration in the context of the Oedipal complex.

The first move (projecting the Voice onto the penis) contracted the high-low topography onto a single surface. But the result was symbolic castration — a splitting of this surface into skin and phantasmatic surface. The second surface, the surface of phantasm or sense, is derived from the phallus as *paradoxical element*, as the attempt to make present that which is unpresentable. It issues from the crack/scar of this failed stitching of the Voice onto the penis, from the mark of castration as connecting the surface of the skin to that of phantasm. But this second surface grows out from this crack to become a separate surface.

From the integrated surface of the body, which betrayed the withdrawal of the Voice, was derived a second surface, which respects this withdrawal. This splitting into embodied intentions and phantasmatic consequences is precisely the structure of language. The resonance of the surfaces allows for the dimensions of language to emerge. The resonance between the embodied surface and the surface of the phantasm generates the first language. The patches of body surface, as elementary phonemes, are chained together under the rule of the organising phallus to form morphemes and semantemes. This surface resonates with the topography of the Voice's pre-sense — the dimensions of denotation, manifestation and signification already articulated by the Voice. This resonance between phantasm and the body is a first language.

It is a question of a dual surface effect ... **which precedes all relations between states of affairs and propositions**. The fact that the second, phantasmatic surface emanates from the crack of castration, the projection of the Voice onto penis, **is why when** the phantasmatic **surface is developed with different effects which at last found denotations, manifestations and significations as ordered linguistic units** (which, we recall, emerged from the heights of the Voice), **elements like phonemes, morphemes, and semantemes** (which, we recall emerged from the sexual surface of the skin) **seem to turn up on this new plane, but seem to lose their sexual**, or embodied, **resonance**. This **resonance is repressed or neutralised, while the** underlying embodiment is **swept aside by the new** topography. **Sexuality** and embodiment exist **only as an allusion, as vapour or dust, showing a path along which language has passed, but which it continues to erase like**

so many extremely disturbing childhood memories (Deleuze 1990, 242).

Language and sense are effects of a paradoxical element, the attempt to impose the withdrawn Voice on the present penis, of a non-sensical binding together of good intentions and tragic results, an image of unity which dissolves itself into two.

To wrap up this digression, we must relate the two myths — the psychoanalytic myth of the generation of language and the myth of the language of pre-crisis mathematics. I will now point out the paradoxical element, that which takes the role of the phallus in tying together pre-crisis mathematics into a doubly-articulated language, a language with phonemes, morphemes and semantemes, with denotation, manifestation and signification — a surface of sense hovering over bodies and things.

3.4 The paradoxical object and the full language of pre-crisis mathematics

Note, however, that we're not going to tell a story of how a mathematical language was generated. pre-crisis mathematical language is already given as established language, and we're not going to provide it with a creation myth. What we intend to do is find in this language the de-sexualised traces of the creation myth above: a place holder for a phallus, a paradoxical element that is structured as the Voice projected onto the penis.

It is in fact remarkably easy to find a paradoxical element, a trace of the phallus, around which pre-crisis mathematics splits into two surfaces that are tied together into a linguistic complex. In fact, there are many such elements; but here we will consider as a canonical example the very element that instigated the Cantorian crisis: infinity.

To explicate in what way the infinite is paradoxical, we must first provide criteria to recognise the paradoxical object, which so far we only know as the phallus, that superposition of the withdrawn Voice on the bodily penis. These criteria are borrowed by Deleuze from Lacan, and underlie the paradoxical element's capacity to relate and split words and things, facts and their senses. The paradoxical object is **missing always its own**

equilibrium, at once excess and deficiency, never equal, missing its own resemblance, its own identity, its own origin, its own *place*, and always displaced in relation to itself. It is floating signifier and floated signified, place without occupant and occupant without place, the empty square (which can also create an excess through its void) and a supernumerary object (which can also create a lack by being this excess number). This is the very object **which brings about the resonance of the two series**, words and things, which it relates (Deleuze 1990, 228).

It almost seems that this description was tailor made for the concept of infinity, as it stood in Zeno's paradoxes, early calculus, and in Cantor's theory of cardinals. It is **at once excess and deficiency**. **Excess**, because it is un-containable, it keeps advancing, whether as an unlimited series of steps, of subdivisions, or of the uncontainable hierarchy of ordinals and cardinals. **Deficiency**, because whenever it seems to be encountered, there is still something missing, a further step, a subsequent subdivision, another 1 to add, a greater cardinal ahead.

Infinity is **never equal** to itself, **missing its own resemblance**, because, on the one hand, it is equivalent to a proper part of itself, without, on the other, being equal to this part. The integers can be mapped one-to-one onto the proper subset of even integers; the infinitesimal dx bears an ambiguous relation of dis/similarity to $\frac{1}{2}dx$ and d^2x. It is **missing its own origin**, because the sequence of integers (negative and positive together) does not have an origin. Even the sequence of steps in the race between Achilles and the turtle, which does have an origin (two origins), fails to reach a final destination (tracking the argument backwards would undermine the origin as well).

The infinite cannot be ***place***d. The infinitesimal fails to have a position between the point and extension; nothing can capture and tie an infinite sequence to a concrete place, it always escapes the place and disappears towards the eventual or metaphysical. The infinite is **always displaced in relation to itself**. This is true for Hilbert's Hotel[7] as it is true

[7] A hotel with infinitely many rooms is in full capacity. One evening, infinitely many new guests arrive. The hotel manager resolves the crisis in the following manner: first she sends each guest from their room, room number x, to room number $2x$; then the newly

for its early echo, Zeno's stadium paradox.

The infinite is a **floating signifier**, because it denotes no present image or state of affairs. It is a **floated signified**, because no chain of words, however elaborate, can fully capture the infinite. It is a **place without occupant** or **empty square**, because no concrete image or object can 'fill the shoes' of the infinite or check the 'I am infinite' box. This void articulates the infinite as an **excess** beyond any concrete image or object. No **place**, name or number can be assigned to the infinite, as some order of infinity always exceeds them. The result is an effect of **lack**, something missing in our reservoir of places, numbers, and names.

This paradoxical element, in the form of a hypothesised pre-existing, complete but withdrawn object, is projected upon the 'body' of applicable mathematics with the good intentions of organising it, completing it, unifying it, resolving its open questions. Due to this projection of the image of infinity, the discrete-becoming-continuous is replaced by the continuous, the small-becoming-smaller and the large-becoming-larger are replaced by the infinitely small and infinitely large respectively. We obtain the full infinite line as well as the dimensionless point. It allows different zones of mathematical language (algebra, geometry, analysis) to be unified under Cartesian analytic geometry, which later develops into algebraic geometry, the field that came to dominate 19^{th} century mathematics. And the movement of foundations seeks to found all mathematical objects on the infinite sequence of positive integers as its final common ground.

But the infinite paradoxical object, like the phallus in the psychoanalytic myth, ends up castrating the body of applicable mathematics, demonstrating a lack within it, and destroying the intended prospects of making mathematical reasoning concretely present. It opens up a 'metaphysical' mathematical surface, a surface where the operation of linguistic effects in discussing the mathematical infinity carry out their own deployment, follow their own logic, with only a faint echo of 'underlying' or 'applied' situations. This metaphysical surface echoes the surface of 'real world' mathematics, but is at the same time not completely dominated by its constraints. Zeno's paradoxes, the Pythagorean crisis, Berkeley's critique of calculus and the

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arrived guests are assigned to the odd-numbered rooms, which have now all become vacant. Everyone goes to bed content.

paradoxes in Cantor's theory are all perfect candidates for an explicit manifestation of this process.

However, since the fourth dimension of language (sense, the expressed, that which is not reducible to denotation, manifestation and signification) will be articulated as insisting only in the proposition, and since we do not have a concrete set of propositions for our mythically reconstructed pre-crisis mathematical language, the task of concretely explicating the effects of sense in pre-crisis mathematics beyond the indication of some paradoxical elements is unmanageable. Our indication of the crucial role of the infinite paradoxical element in the constitution of pre-crisis mathematical language will therefore be demonstrated by our diagnosis of the 'depressive' state of mathematical language that resulted from the finitist-formalist attempt to remove this paradoxical element. We will go on to describe explicitly Gödel's 'therapy' for this mathematical depression, and then, finally, turn to describe the operations of the fourth dimension of sense as they insist in contemporary mathematical language.

3.5 First finalé: Gödel as Lacanian therapist

My task here is to eventually point out the role of a fourth dimension of sense in Gödel's mathematical language. But I will first follow the attempts to extract and discard the paradoxical element from mathematical language, go on to point out the tragic consequences of this well-intended attempt, and finally reconstruct Gödel's intervention as a therapeutic move that would restore to mathematical language its fullness (or, rather, openness).

Benacerraf and Putnam point out to an **ideal, shared by thinkers with views as mutually antagonistic as those of Hilbert and Brouwer, of eliminating the infinite from mathematics altogether** (Benacerraf & Putnam 1983, 6). The finitist-formalist attempt reduced the mathematical infinite to a finite **system of symbols together with rules for employing them** (1934, 346). When thus reduced, the following image of mathematical practice emerges, as painted by von Neumann in 1931: **even if the statements of classical mathematics should turn out to be false as to content, nevertheless, classical mathematics in-**

volves an internally closed procedure which operates according to fixed rules known to all mathematicians and which consists basically in constructing successively certain combinations of primitive symbols which are considered "correct" or "proved". This construction-procedure, moreover, is "finitary" and directly constructive (von Neumann 1983, 61–62).

The resulting mathematical language shrugs its shoulder at the prospects of false statements (**even if the statements of classical mathematics should turn out to be false as to content** ...), as long as the mathematical procedure is sound. Statements are no longer true or false, but only provable or unprovable. The rule of denotation is thus crossed out from the horizon of the mathematical language. Manifestation too loses its footing. The formal language has no symbol for 'we'. One may point out to the axioms as forms of manifestation; however, the possibility of repositioning axioms as assumptions, turning each statement into a conditional statement assuming the axioms, transforms axioms into a very weak form of manifestation, if at all.

The only dimension left is signification. Formal languages are subject to machines (or rules), which determine implication, which analyse provability conditions, and which separate meaningful symbol combinations from meaningless ones. All rules must be finitely and mechanically verifiable. The resulting language is one dimensional. The chains of symbols in the language are subject to a dimension of signification, but this dimension is not floating above — it is tied to the ground, just as in the attempt to anchor the Voice to the surface of the body.[8]

This kind of language is a sort of concrete analogue of the dimensionless language on the surface of the skin: symbols (phonemes), syntactically organised into chains (morphemes), and related to each other by rules of inference, which allow syntax to feign a flat, reduced, semantics. But above and below this surface there is nothing, and this surface does not even manifest itself. This stupid surface relates only to the syntactic machines

[8]In this context one may describe intuitionism as an attempt to reduce mathematical language to a single dimension of manifestation: **The intuitionist mathematician proposes to do mathematics as a natural function of his intellect, as a free vital activity of thought** (Heyting 1983, 52). Logicism would lie a little closer to a two dimensional language displaying signification and, to a very limited extent, denotation.

operating on it. It does not express.[9]

Any contemporary mathematical argument can (**in principle**) be transcribed into this language. This includes, of course, Gödel's text. Such readings seek to view mathematical texts from the point of view of the machines operating on the stupid surface. But unless the symbols are allowed to designate (so that numbers designate formulas, rather than just be syntactically related to them), unless they are allowed to manifest (a formal system that has authority over other formal systems, and can perform self-reference), unless they are allowed to signify (not only their own step-by-step validity, but open ended concepts of completeness and truth), Gödel's proof, which would remain a proof, would not be a proof of incompleteness. It will only place a certain chain of symbols at the bottom of a proof. Even the interpretation of this chain of symbols as making an arithmetic claim — the UNPROVABILITY of a couple of integers — which conditions their intended metamathematical interpretation as claiming the unprovability of a formula, would remain inaccessible without a dimension of denotation. If the formally transcribed proof were effectuated and presented as such to a reader, the reader would be nothing but the passive synthesiser of formulas under rules of inference. This reader would never decipher a message of incompleteness in the proof, simply because this reader would never decipher a message. Deciphering messages would be beyond this reader's horizons.

Unless the formal text is immersed in the volume of underlying objects, overlaying concepts, and self-manifesting desire (the indispensable element of motivation in a proof), the proof will remain mechanically correct, but most likely unreadable, and most certainly impossible to understand. Unless someone were to take the elements of the proof and reiterate them into the framework of a full (four dimensional) language, the proof would

[9]The following quotation brought to my attention by Menachem Fisch is quite appealing here. 19^{th} century mathematician and logician Augustus De Morgan claims that a person **who makes the transformations of algebra by the defined laws of operation only is comparable to one who puts a dissected map together by the backs of the pieces alone; whereas the person who looks at the front, and uses his knowledge of geography to help, more resembles the investigator and mathematician** (De Morgan's 1842 *On the foundation of Algebra* quoted in Fisch (1999, 148)).

be nothing but a dead letter.[10]

This is the point where Gödel intervenes. His restoration of fullness and sense to mathematical language is threefold. First, Gödel reminisces. **While a formal system consists only of symbols and mechanical rules relating to them**, he explains, **the meaning which we attach to the symbols is a leading principle in the setting up of the system** (1934, 349). Gödel reminds us that even if we find language in this state of formalist depression, this language must have had a healthy history of full livelihood. Gödel's first move is to reconstitute a past. In this past, which Gödel is about to reconstruct, there shall be **meaning**, **expression**, **signif**ication, **denot**ation, and the entire multidimensional transcription mechanism reviewed in the first chapter of this book.

This to-be-reconstituted past will also be drenched with a manifesting **we**, and even include a **desire** (to forget). This is not the past of a recollection, a re-presenting of something that was present at a now-gone-but-once-present moment. Gödel refers to no concrete moment; his formal system is one of unboundedly many variations circulating at the time. The past to which Gödel refers is re-produced ad-hoc, as in nostalgic reminiscence. Instead of representing a past-present as a present-present it seeks to recreate a new surface of active synthesis. But the origin, the attached **meaning** in the **setting up of the system** does not restore the specific history of a given language. Instead, it hypothesises a visionary past, and reforms an entire history of formal languages from Boole to Hilbert, drawn through radical attachments and detachments of texts and meanings to a horizon of eternal return.[11]

The product of Gödel's reminiscence, his recollection of a past that never took place and that opens the road for new futures, his multiple articulation of meaning as leading principle, as mechanical verificationability

[10]Of course, such reiteration needn't be compatible with Gödel's framework. The elements of the 'dead' formal proof are open to outside interventions and interpretations, which may read into the proof various mathematical and/or non-mathematical meanings (Gödel's numerological coding of formulas is not the only numerology that the text can sustain).

[11]See Deleuze's notes on Proustian reminiscence in Deleuze (1994, 122), and, once again, Derrida's reading of Husserl's *Origin of Geometry* (Derrida 1989) as presented above.

of formal rules, and as something forgettable that can be detached from and attached to texts — all this constitutes a paradoxical element, which ties the language and its meta/outside while maintaining their distinction, and, in turn, as we shall see, opens up an entire dimension of sense.

But what comes to substitute the lost paradoxical element in Gödel's move is not meaning. His choice is much more bold, perhaps even shocking. He dares summon the high object par-excellence, a variant of the psychoanalytic Voice. He summons truth to play the role of the paradoxical element.

Truth, we recall, has lost its bearing on the formal system together with the discarding of denotation. What was left behind was but a syntactic determination of provability. Gödel summons truth, but as distinct from provability. According to his formulation **"false statement in** the language B**" cannot be expressed in** B, and the same goes for the predicate 'true' (1934, 363). The truth predicate of any given language must be expressed in yet another language. In his construction, therefore, truth, like the phallus, is **excess** (the truth predicate of a language always exceeds that language) and **deficiency** (no articulation of truth can rule over the full hierarchy of languages). Truth predicates keep changing between languages, having no stable **place**, self-**resemblance** or **identity**. They are **displaced** from language to language, so they have no **origin**. No predicate can exhaust truth, and truth cannot be fully formulated, leaving it on the one hand a **floating signifier** and **empty box**, and on the other a nameless object and **floating signified**; at once **excess** of demand and **lack** of fulfilment (as well as the opposite: **excess** of possible truth predicates, and **lack** of a complete verification procedure).

With this paradoxical object the surface of language is not just doubled, it proliferates unlimitedly, generating an infinite hierarchy of bifurcating language surfaces, restoring through this hierarchy the infinity that had been previously excluded. Each surface is at once a full language doubled by another surface of the sense insisting in its propositions. In relegating truth to the position of paradoxical element, the fullness of mathematical language has been reinstated under the splitting law of castration, and the past one-dimensional depression is healed. This is, perhaps, why Gödel's proof did not create a mathematical crisis. It did not create a crisis, because

it was a therapy, rather than a pathology.[12]

But this still is not enough. Some mathematicians and logicians accept Gödel's proof, but reject his concept of truth. They nevertheless enjoy access to a full, four dimensional, mathematical language. This is so because the paradoxical element is reintroduced not only in the form of truth, but also in the form of self-reference, which Gödel restores after it had been discarded by Russell and Whitehead. To clarify this point, let's read more about the structure of the paradoxical object. **It is a two sided entity, equally present in the signifying and signified series. it is the mirror. Thus, it is at once word and thing, name and object, sense and *denotatum*, expression and designation, etc. It guarantees therefore, the convergence of the two series which it traverses, but precisely on the condition that it makes them endlessly diverge. It has the property of being always displaced in relation to itself. If the terms of each series** (signifier and signified) **are relatively displaced, *in relation to one another*, it is primarily because they have in themselves an *absolute* place; but this absolute place is always determined by the terms' distance from this element which is always displaced, in the two series, *in relation to itself*. We must say that the paradoxical entity is never where we look for it, and conversely that we never find it where it is. As Lacan says,**

[12] We should perhaps account for our use of the term 'Lacanian' in the title of this section. The obvious reason is, of course, our heavy reliance on Lacanian concepts, mediated through Kristeva and Deleuze. But there is a little more to it than just that. The early texts of Lacan can be read as suggesting that allowing the signifier to exhaust the positions it can occupy under the laws of symbolic order is a road to mental health. For example, **little Hans, left in the lurch at the age of five by the failings of his symbolic entourage, and faced with the suddenly actualised enigma to him of his sex and his existence, develops — under the direction of Freud and his father, who is Freud's disciple — all the possible permutations of a limited number of signifiers in the form of a myth, around the signifying crystal of his phobia. We see here that, even at the individual level, man can find a solution to the impossible by exhausting all possible forms of impossibilities that are encountered when the solution is put into the form of a signifying equation** (Lacan 2006, 432). To an extent, this is precisely Gödel's operation: exploring permutations and forms of impossibilities around the kernel of self-reference or truth, in order to escape a fixation that immobilises mathematical language, and prevents it from functioning along all its dimensions.

it is placed out of place (elle manque à sa place) ... **It belongs to this element which is always absent from its proper place, proper resemblance, and proper identity to be the object of a fundamental question which is displaced along with it: what is the Snark? what is the Phlizz? What is This *(Ça)*?** (Deleuze 1990, 40–41, 57; translation modified).

Let's begin with **This** then. **This** has been the focus of many a philosophical debate. In these debates its status ranges between the most distinguished model of denotation and its most defective specimen. Its ambiguous role is not restricted to the occasion of the liar's 'This statement is false'. **This** is also the paradoxical element of Magritte's **This is not a pipe** (see Foucault 1983): **This** which may or may not point to the diagram (which may or may not be a pipe or a drawing of a pipe), and which may or may not manifest itself (*that* is indeed a pipe, but **This**, *this* word, **This**, is certainly not). Gödel installs **This** at the core of his argument. $S(w,w)$ is **This**.

Let us recall again the construction of Gödel's undecidable formula. $\Pi v[\neg B(v, S(w,w))]$ is shorthand for the formula (numbered p), which says that the formula, whose number is $S(w,w)$, is not provable. Recall also that $S(z_a, z_b)$ is defined so as to yield the number representing formula number a, after z_b (the numeral corresponding to the number b) is substituted for the free variable w.[13]

Here, the undetermined $S(w,w)$ is as undetermined as **This**. It has the potential to refer to many different formulas, to numbers, and, most importantly, to itself. If we substitute $S(z_p, z_p)$ according to the above rule, we obtain the number of the formula $\Pi v[\neg B(v, S(w,w))]$ with z_p substituted for w. In other words, we obtain $\Pi v[\neg B(v, S(z_p, z_p))]$. This formula says of itself: This statement is unprovable. Constructing this **This** with the limited tools of a depressed formal system, installing this self-referring mirror inside this formal language, this is Gödel's most overwhelming achievement.

To observe that $S(z_p, z_p)$ can take the place of the paradoxical element, note that it is both **signifying** (a number) and **signified** (by a numeral), it is at once **mirror** (reflecting itself as its own subject), **name** (of a formula), **thing** (a sign sequence), **word** (on the page), **object** (of study),

[13]See section 7 of Chapter 2 for more detail.

sense (the undecidable formula, the collapse of Hilbert's programme), ***denotatum***, **expression** and **designation**. Being all at once, it guarantees the **convergence** of series. Through this element the language is now bound together with what it denotes. The language subsumes itself into its objects. $S(z_p, z_p)$ keeps **displacing** itself, relating to itself as if it were something else, which can again relate to itself as an other in turn.[14] This is the **absolute** core element around which, for which, the argument is drawn. Everything else in the argument is **relatively** placed with respect to **This**, despite the claim that its self-referential structure was discovered **so to speak by chance**.

And, as we have already established in the previous chapter, it is **never where we look for it** — it is never properly written down, except in the form of shorthand, denotation, representation; again and again we are told that **in principle** we can compute it. And yet, it is never considered a fault that the computation is never conducted; that it may be materially impossible to conduct it; that it would be unreasonable for any human to conduct it, or even for a human to verify its implementation by a machine; that its presentation necessarily depends on deferring authority of judgement to an indefinite, hypothetical, **in principle**.[15]

At the same time, by providing us with an extreme test case, it allows to maintain the regular cases of **diverge**nce between the series of number and formula, formula and meaning. Relative to $S(z_p, z_p)$, the 'regular' case of $S(z_a, z_b)$ is easy to handle. There we know — not without problems, but much more clearly than for $S(z_p, z_p)$ — which numeral means number (z_b), which numeral means formula (z_a), and which refers to which in what order. In fact, it is through this relative clarity that the self-referential structure

[14] I refer here to the analysis in section 7 of Chapter 2, specifically to the discussion of the nested substitution of $S(z_p, z_p)$ into itself.

[15] I leave it as an exercise to repeat the analysis of Gödel's articulation of truth in these terms, and to demonstrate how **"... truth joys** (jouit). **Truth is not the conclusion of a system, truth is joying. There are people who go looking for truth, it doesn't have to be looked for, it just comes; it can be found in bed, but how does one get it into bed? By talking of other things, as so often happens. By talking of other things, not philosophy, not truth, but something else. It is then frequently seen as a good in itself: in effect, it is often covered in veils, like a bride. Above all, one must refrain from using it until after it has joyed — please excuse all this..."** (Derrida quoting Francis Ponge in Derrida (1984, 94)).

of $S(z_p, z_p)$ can be narrated as successive rather than as circular, and said to be discovered **so to speak by chance**.

Indeed, many readers follow Gödel in denying the circularity of self-reference and in providing revisionary accounts. **Contrary to appearances, such a proposition involves no faulty circularity, for initially it [only] asserts that a certain well-defined formula (namely, the one obtained from the qth formula in the lexicographic order by a certain substitution) is unprovable. Only subsequently (and so to speak by chance) does it turn out that this formula is precisely the one by which the proposition itself was expressed** (1931, 151). Many readers indeed would deny the conjunction of functions I imposed on $S(z_p, z_p)$ above, and claim that it can fit correctly only into a few of them. But such a revision and restrictive reading can only be produced after the fact. This is the lightning of representation having erased the trail of the dark precursor, which had conjured it. Nothing happened here by **chance**, and no narrative or transcription of the proof can prevent the reader from passing through the mirror of $S(z_p, z_p)$ and finding there the conflation of several layers and dimensions of language. I have never met, and cannot imagine, the person who would understand Gödel's argument as a 'lucky monster' produced from the chance assembly of symbols monkey-typed in a one dimensional language.

Which doesn't mean that there is anything wrong in revising the narrative this way. And there is nothing wrong in manipulating someone to understand the argument as a 'lucky monster' by way of a clever contrived experiment. The paradoxical object does not rule over the text. The text is inherently subject only to what comes from the outside: iteration and revision. No one can protect the text from being quoted away from its circumstances, and no one need try. The law of the paradoxical object is to allow its selfless self to disappear. This is its essence. If we occasionally betray it, it is only to induce the ensuing effects of castration: the emergence of a new phantasmatic surface.

And yet, wherever something makes *sense*, the paradoxical object will have been found, provided we maintain a right for looking. **Whether we ought to be content with these three dimensions** (denotation, manifestation, signification) **or whether we should add *a fourth — which***

would be sense **is an economic or strategic question. It is not that we must construct an a posteriori model corresponding to previous dimensions, but rather the model itself must have the aptitude to function a priori from within** (Deleuze 1990, 17). But this strategic choice does not exclude a positivist, empiricist, or Wittgensteinian critique that would dismiss this text and my reading as pure nonsense. Sense and nonsense are not antagonistic; rather, they cross-constitute each other, as would be the case were such critique to claim that a sensible view should cross out and cover over my nonsensical discourse.

The claims of the discourse of (non)sense cannot be properly proved. They are posited as **a priori**, and posited in a way that guarantees that they cannot be refuted — indeed, dark precursors are, by definition, there to be effaced. The discourse of (non)sense is there to answer an **economic or strategic** concern. This **strategic** concern that constitutes this discourse is, I believe, a concern with enabling intervention into established discourses — a strategic instrument that would respect, to an extent, the discourses' stated limitations, but permit them to link via postulated constitutive nebulae to outside interventions, and, just as importantly, permit them to discard such former interventions. The discourse of (non)sense is therefore a discourse designed for the reconfiguration of responsibility. This discourse of (non)sense is necessary to the extent that every text has undergone some sort of outside intervention 'before' it 'began' to denote, manifest and signify; it is necessary to the extent that any discourse can 'desire to forget' any given intervention; it is necessary to the extent that such interventions and desires can be *reconstructed* as 'primordial' or 'eschatological' nonsense. But we never claim that (non)sense ever necessarily *is*.

Which brings us to the moment where we explicate the dimension of sense. Not through its mythically contrived generative narrative, nor through paradoxical objects that may or may not, will or will not, have been produced in hindsight in order to be pointed out as that which was effaced in order for the language to function and for its surface to arise. Now is the moment to observe sense functioning *in* the mathematical text.

3.6 Second finalé: Does Gödel's text make sense?

Sense is the event expressed by the proposition and insisting in it. As an independent surface it has its own motility, which is not subjected to the other logical dimensions of language. It is not about true and false denotations, veracious and illusionary manifestations, or possible and absurd significations. It operates by crossing out these determinations and holding on to what is left, what is left insisting in the words.

I am now taking back the statement that I made at the opening of this chapter, which said that sense did not come between words and actions, propositions and the body, speaking and eating; it does. But when we say 'add 1', Wittgenstein is right to emphatically deny that we first attain the sense of the decree, and then perform the actions. Sense is not a stage between words and actions. Sense is the surface that separates words from action, and that allows them their respective in(ter)dependence, a form of restricted relative dependence that Wittgenstein need not deny. Sense is not a station on the way from words to actions. It is, rather, **the frontier, the cutting edge, or the articulation of the difference between the two terms, since it has at its disposal an impenetrability which is its own, and within which it is reflected** (Deleuze 1990, 28).

What is this surface of events, which insists in words, but is not yet effected in action? In 'natural' language, according to Deleuze, it can be found in the infinitive form of the verb, that which is not yet temporalised, not yet effected in a subject or a pronoun. In formal languages it is even more straightforward to find sense. It can be found in the predicate operating on a free variable. In the string $P(x)$ insists the predicate P unsubjected to a quantifier and uneffectuated by substitution. As such it doesn't designate, and can't be true or false; it does not manifest any consciousness, desire, belief or causality, and cannot be veracious or illusory. This holds for Gödel's text as well as for any formal language.

But formal language, which is concerned with covering over the unstable operation of sense, quickly reacts. In Gödel's text we read that an **expression A in which the distinct free variables $t_1, \ldots, t_n$ occur shall mean the same as** $\Pi t_1(\ldots(\Pi t_n(A))\ldots)$ (1934, 352) (Π here is the

universal quantifier). In order to yield to the order of signification, a free variable is assumed to be bound by a universal quantifier. Even where this semantic rearticulation is not explicitly stated, it is standardly implicit in rules of inference that derive $\Pi x A$ from A (this is why I cannot claim that $P(x)$ does not signify — even as it stands it is already involved in an inferential relation).

In formal logic, according to the rule quoted above, the variable is never free. Unless otherwise determined, it represents the universal. But sense insists in formal languages to the extent that the free variable, for the brief period between the moment it is written and the moment it is bound by a quantifier or subjected to an inference rule, during that small portion of text, it is still free. This empty point in time is the time of the pure event, of the pure unsignifying predicate. This moment is crossed over by the rules of formal logic. But formal logic could not exist without this moment. This moment insists in the language. To decide whether the variable itself (or the predicate operating on a free variable) is a paradoxical element, one is required to decide how to read it. Is it read during the ever splitting, centrifugal time, in which it makes sense (the time which Deleuze calls **Aion**), or is it absorbed by the sequential and centripetal time of the argument?

In the Fifth chapter of *The Logic of Sense* Deleuze presents four 'paradoxes' where sense is reflected and **developed for its own sake** (Deleuze 1990, 28). We will follow these paradoxes to demonstrate how the predicate operating on a free variable manifests the neutral motility of sense.

The paradox of regress or of indefinite proliferation. According to this paradox, **I never state the sense of what I am saying. But on the other hand, I can always take the sense of what I say as the object of another proposition whose sense, in turn, I cannot state. I thus enter into the infinite regress of that which is presupposed. This regress testifies both to the great impotence of the speaker and to the highest power of language: my impotence to state the sense of what I say, to say at the same time something and its meaning; but also the infinite power of language to speak about words** (Deleuze 1990, 28–29).

I can never write the sense of $P(x)$, the event of its writing as not yet

actualised, not yet subjected to denotation, manifestation and signification — I can only write $P(x)$ and let its sense insist in what I write. But if the formal language is allowed the liberties of higher types, the writer has the privilege of predicating over predicates, and over variables standing for predicates (if not, then the meta language has this privilege). I can write $Q(P)$; now the fact that there is a question whether I predicate over the text P or over what it stands for is precisely indicative of the lining of sense in this predication, distinguishing signs from what they express. Either way, $Q(P)$ is not its own sense. It expresses a sense, which we may go on talking about (predicating over Q), regressing indefinitely.

In a similar way, according to Gödel (together with Tarski and Carnap) each language has a truth predicate, which is written in another language, another language which rules over the one whose truth we are considering. The language where the truth predicate is formulated is ruled over, again, by a truth predicate formulated in yet another language. And so on. We speak one language at a time, express one formula at a time, and can never express at the same time a statement and its sense or its truth predicate. But we may go on forever speaking about the sense and truth predicates of what we expressed.

The paradox of sterile division, or of dry reiteration. **There is indeed a way of avoiding** the above **infinite regress. It is to fix the proposition, to immobilise it, just long enough to extract from it its sense — the thin film at the limit of the things and words** ... **As an attribute of states of affairs, sense is extra being. It is not of being, it is an *aliquid* which is appropriate to non-being. As that which is expressed in the proposition, sense does not exist, but inheres or subsists in the proposition** ... **The two paradoxes, that of infinite regress and that of sterile division, form the two terms of an alternative: one *or* the other** (Deleuze 1990, 31–32).

Above we traced an analysis of $P(x)$, as a predicate operating on a free variable, during the empty time where it is free from denotation and manifestation. This analysis was in fact inspired by the formulation of this second paradox. If we observe $P(x)$ inside this form of time, what we grasp there is its sense — the boundary between the written letters and what we may do with them: designate, manifest, signify, and generally obey their

orders. But this moment of separation is also that which allows us to do with the text something *else*, to actualise differently the event of its writing, to inscribe it in an order not yet specified or anticipated.

But all this may be too obscure. In Gödel's text the invocation of sense as the sterile double of the text appears more visibly when numbers, for instance, are assigned the role of symbols in a formal system, but as such that **of course, cannot be arranged in a spatial order** (1931, 147). Here we encounter the sterile, unwritable numbers, which hover above the concrete symbols and concrete number representations. This moment of abstraction is the transition between the concrete letters of a formal text, and the denotations, manifestations and significations to which they will give rise once they will have gone through Gödel's double encoding (of formal text as numbers and of properties of numbers as formal expressions).

We must not confuse, however, these sterile numbers of the lining of sense with ideal, platonic numbers. The latter are subject to the reign of truth, veracity and signification as elements in a distinct order relation and arithmetic structure. This is why Gödel's argument, in order to apply to formal systems, must concern itself with sterile numbers. In order for the argument to function, the numbers, which are no longer effectuated as physical scribblings on paper, must — and this is crucial if we are to accept these numbers as arbitrary elements of a formal system — not yet be immersed in the sequential algebra of the positive integers, which denotes phenomena in the world, manifests the form of the unbounded sequence, signifies in the structure of arithmetic, and is generally much too charged to function as the arbitrary symbols of a formal system.

But our sterile numbers are distinct from platonic numbers in yet another important way. Platonic numbers fail to enjoy the motility conferred upon sterile numbers by the changing texts and contexts at the verge of which sterile numbers hover. This is the motility that allows sterile numbers to stand for so many different things (formulas, numerals, statements) — things that platonic numbers cannot be confused with. This instability is precisely what separates the motion of eternal return from the stability of ideal origin. When we extract an ideal object from a text, we extract the *same* object every single time. When we extract the sense insisting in the proposition by discarding its material bearer (namely, every time we

quote a text) we do something *different* at each and every instance (we re-contextualise differently every time we quote). Every time we quote, we have the opportunity to fold the second, phantasmatic surface between the surfaces of text and action in a different way. The distinct forms of materiality constrain the extractions of sense and make of them events affirming difference. Only then can we start imagining active and passive syntheses of repetitions as ruling over such extractions.

The paradox of neutrality, or of essence's third estate ... **Sense is strictly the same from the point of view of quality, quantity, relation, or modality. For all of these points of view affect denotation and the diverse aspects of its actualisation or fulfilment in a state of affairs** ... **"God is" and "God is not" must have the same sense, by virtue of the autonomy of sense in relation to the existence of the *denotatum*** ... Sense is **indifferent to all opposites. This is so because all of these opposites are but modes of the proposition considered in its relations of denotation and signification, and not the traits of sense which is expressed** (Deleuze 1990, 32–35).

Again, we point to the analysis of the free $P(x)$ above. Lacking a truth value, the free $P(x)$ and the free $\neg P(x)$ both express the sense of either. In Gödel's text, this is precisely where the **desire** that meaning-attachment **be forgotten** comes in. The text is stripped of all modalities, qualities, quantities, and relations. Yet it still makes sense. It makes sense as an event of writing, an event to be coded under the mechanism of formula enumeration. Indeed, a formula and its negation will be encoded by different numbers, but it is not because they are a formula and its negation that they are coded differently. The coding of formulas as numbers does not mind the different significance of, say, $\neg$ and $\forall$; it can easily exchange the two and continue to serve effectively. It can also indifferently exchange the coding of $P(x)$ and $\neg P(x)$.

But it is possible to make this point in other ways. The undecidable proposition, which escapes, together with its negation, the determination 'provable', manifests, for a brief moment, its sense independently of the semantic machine. Both the formula and its negation reflect in each other a single event; they both express the same moment of undecidability. A sim-

ilar manifestation of sense occurs with the inference rule described above, which allows to add a universal quantifier to a provable proposition with a free variable in precisely defined syntactic circumstances. Here, again, sense insists in crossing over the boundary of the universal/particular dichotomy.

These manifestations of sense do not cross out the differences between the undecidable proposition and its negation, or between a statement with or without a universal quantifier; on the contrary, the differences established through the link of sense cross over their common sense. These differences may be articulated in many ways. But without this sense, without the ability to redistribute universality and particularity, there would not have been any link between particular and universal that would allow the former to imply, under certain conditions, the latter; without sense there might be mutual exclusion, but not a *relation* of mutual exclusion *between* a formula and its negation.

The paradox of the absurd, or of the impossible objects ... **the propositions which designate contradictory objects themselves have a sense. Their denotation, however, cannot at all be fulfilled; nor do they have a signification, which would define a type of possibility of fulfilment. They are without signification, that is, they are absurd. Nevertheless, they have a sense**. Such objects include **square circles, matter without extension, *perpetum mobile*** ... **If we distinguish two sorts of beings, the being of the real as the matter of denotation and the being of the possible as the form of significations, we must yet add this extra-being which defines a minimum common to the real, the possible *and the impossible*** (Deleuze 1990, 35).

Nothing prevents the formal appearance of $P(x)\&\neg P(x)$. Nothing even prevents the meaningless $PP((x$ from appearing. The latter is discarded by the formal system, but only after the system states rules that acknowledge it as something we must discard. But neither the contradictory nor the meaningless are excluded from Gödel's enumeration. Beyond actuality and possibility, even those strings of symbols that are not **meaningful formulas** participate in the enumeration of formulas. Gödel's text allows the absurd to be processed by its machines.

And to the list above of terms in Göodel's mathematical language

without formally determinable universal designation, signification and possibility of fulfilment we must add, according to Gödel's own rearticulation, one more term: Truth. Surely truth, truth which is beyond any specific language, has some sense to it.

It is under the common regressing and sterilising lining of sense, or, rather, through it, that the dimensions of denotation, manifestation and signification function. The resonance that effectuates denotation (resonance between words and bodies), manifestation (resonance between words and consciousness), signification (inferential resonance between words and words), this resonance depends on the possibility of resonance itself. It depends on a paradoxical element that renders separate the two, which are supposed to resound each other, first and foremost by separating itself from itself. It depends on a milieu, on the *between* across which the series resonate, a sterile film without which there will be no separation of the two which resound, but only a rumbling of partial objects or the stillness of the catatonic body without organs. A film, whose only property is setting apart by conducting resonance. A resonance in itself. A hymen.

I do not claim that this spacing must be filled or accounted for. This is simply the doubling across which intervention, interpretation, transcription *can* take place. But they don't *have to* take place. Indeed, **Explanations come to an end somewhere** (Wittgenstein 1953, §1). And Wittgenstein is right in juxtaposing the moment where explanations come to an end to the moment where someone **'acts'** and the moment where something is **used** (this is why, perhaps, he makes a point of invoking the **end** of **explanations** already at the very first aphorism of the *Philosophical Investigations*). Eventually, we give up this doubling, where intervention, interpretation, transcription can take; we cross over it, make the quantum leap, no holds barred, no questions asked. But **explanations** needn't **come to an end** *right now*. This dis-joint, the potential to go on explaining further some other time, somewhere else, allows us to cut-and-paste, re-edit the film, re-fold the hymen, interpret further, and end up elsewhere, where we do other things with our words.

There is never a moment of final articulation, even where, just like here, explanations come to an end.

3.7 Third finalé: Revolution in mathematical language?

In her *Engendrement de la Formule*, the closing paper of *Semeiotike* (which appeared in the same year as Deleuze's *Logic of Sense*), Kristeva lays out some elements, which appear to correspond to Deleuze's surface of sense. The infinitive form of the verb receives special attention, and is identified as **a scene of signification where that which is accomplished is not yet, because it is in the process of being. We are therefore facing a modality of signification that designates a generation escaping time, escaping "situation" and "narration", having no beginning nor end, neither subject nor recipient, but forming itself in a growth which, in order to underpin achievement and beginning, obtains the value of a *rule*, of an *order*, of a *law*, for which the subject and his temporal and personal modalities are suspended** ... **Oriented toward signification as a process of generation, it designates that that which "expresses" itself is a constant becoming, a growth never limited in the time and in instances of speech, but always there, obstinately present, a present being become law, which, at once, is absent both from being and from the present** (Kristeva 1969, 326–327).

Kristeva also brings up a notion of number (the paper we quote is instigated by a reading of Sollers' *Nombres*), which brings to mind Gödel's operation of formula enumeration. This number **is the first movement of organisation, that is of demarcation and ordination. A movement that differs from the simple "to signify" and, we would say, covers a vaster space where "to signify" can be understood and placed**. Kristeva posits an ***infinite enumerator* that disposes of an *enumerated* (graphic or phonic ensembles) before finding it a referent or a signified and turning it into a sign. Mark, knot, ranking, pointing-towards/anaphore/: such are the functions of the enumerator. Sticks, scratches, knots, shells, nuts: such are the first numbers (in the 4th millennium before Christ the Maya already counted by knots and by bundles of ropes). Ranking sticks** ... **is already an arrangement of the infinite and the basis of the system**

of enumeration (Kristeva 1969, 294–295).

But Kristeva is not interested in the project of capturing the conductive lining between the body and words. The infinitive form and the **infinite enumerator** serve as expressive indicators of the semiotic generative process, the *genotext*, which precedes the *phenotext*, namely written and spoken words. her interest is not in the mediation of so-called sense between genotext and phenotext, but in the genotextual process, and the ways by which it secretes the phenotext. The genotext, according to Kristeva, is the unstructured process, akin to that which we formerly discussed under the term 'chora', the **nonexpressive totality formed by the drives and their stases in a motility that is as full of movement as it is regulated** (Kristeva 1984, 26).

Designating the genotext in a text requires pointing out the transfers of drive energy that can be detected in phonematic devices (such as the accumulation or repetition of phonemes or rhyme) and melodic devices (such as intonation and rhythm), in the way semantic and categorial fields are set out in syntactical and logical features, or in the economy of mimesis (fantasy, the deferment of denotation, narrative, etc.). The genotext is thus the only transfer of drive energies that organises a space in which the subject is not *yet* a split unity that will become blurred, giving rise to the symbolic. Instead, the space it organises is one in which the subject will be *generated* as such by a process of facilitations and marks within the constraints of the biological and social structures.

In other words, even though it can be seen in language, the genotext is not linguistic (in the sense understood by structural or generative linguistics). It is, rather, a *process*, which tends to articulate structures that are ephemeral (unstable, threatened by drive charges, "quanta" rather than "marks") and nonsignifying (devices that do not have a double articulation) (Kristeva 1984, 86).

In Gödel's text, therefore, in order to summon the genotext we need to discuss the way semantics is set out in **syntactical and logical features**, and the complex **economy of mimesis**. We have already accomplished the bulk of these tasks in the two preceding chapters. But in order to look

into the genotext we should read the verisimulating syntax, the reduction of semantic determination to syntactical verification, and the elaborate system of **denot**ing, **say**ing, **meaning**, **express**ing, etc. as motions of discrete quanta, which are not yet doubly articulated into signifier-signified, subject-object positions. To reach the genotext we must obey Gödel's **desire** that meaning **be forgotten**.

In a way, the finitist-formalist reduced language is the meaningless linguistic surface, which undoes double articulation, and which arises if we attempt to reverse the effects of what the genotext has produced. As such, this language can only enumerate and count, like rhythmical knots and sticks. Such marks do indeed mark, but they fail to function as signs. The formalist-finitist approach insists on freezing these marks on paper, as an end-product set and done, rather than let it pulsate and conduct the labour of genotext. What separates the dead language of formal logic[16] from living mathematical languages is indicated by Gödel's insistence on building a full, meaningful language on top of the marks, which the semiotic combinatorial process discharges onto the surface of the page. What keeps the formal language dead is an attempt to undo the double symbolic articulation and withdraw from what Kristeva (after Husserl) calls a *thetic phase.*

We shall distinguish the semiotic (drives and their articulations) from the realm of signification, which is always that of a proposition or a judgement, in other words, a realm of *positions*. This positionality, which Husserlian phenomenology orchestrates through the concepts of *doxa*, *position*, and *thesis*, is structured as a break in the signifying process, establishing the *identification* of the subject and its object as preconditions of propositionality. We shall call this break, which produces the positing of signification, a *thetic* phase. All enunciation, whether of a word or of a sentence, is thetic. It requires an identification; in other words, the subject must separate from and through his image, from and through his objects. This image and objects must first be posited in a space that becomes symbolic because it connects the two

[16] This language merits the adjective 'dead', among other reasons, because, while many people are proficient in it, it is never a first language, not even a first mathematical language, for anyone.

separated positions, recording them or redistributing them in an open combinatorial system (Kristeva 1984, 43).

We have already recounted the narrative that extracts from the semiotic chora a realm of signification, going through the mirror (or Voice) and castration. But once established, the thetic position is never completely safe. The unstable motility of the semiotic chora can always transgress the thetic position. **This transgression appears as a breach [*effraction*] subsequent to the thetic phase, which makes that phase negative and tends to fuse the layers of signifier/signified/referent into a network of traces, following the facilitation of the drives. Such a breach does not constitute a positing. It is not at all thetic** ... **On the contrary, the transgression breaks up the thetic, splits it, fills it with empty spaces, and uses its device only to remove the "residues of first symbolisations" and make them "reason" ["*raisonner*"] within the symbolic chain. This explosion of the semiotic in the symbolic is** ... **a *transgression* of position, a reversed reactivation of the contradiction that instituted this very position**.

The proof is that this negativity has a tendency to suppress the thetic phase, to de-syn-thesise it. In the extreme, negativity aims to foreclose the thetic phase, which, after a period of explosive semiotic motility, may result in the loss of the symbolic function, as seen in schizophrenia (Kristeva 1984, 69). This is precisely the pathology of the formal language. An explosion of combinatorial-semiotic motility, which threatened to destroy the symbolic function, together with the volume and inter-surface resonances of pre-crisis mathematical language.

But the alternatives are not clearly-cut along the axis of maintaining or destroying the thetic position. The very motion of transgression, the very motion of rejection, may have productive and healthy consequences. In Gödel's text, something happens with the combinatorial structure, which may be the written correlate of the fact that **rejection**, which is an energetic process at the level of drives, **may pass through the vocal apparatus**. The oral cavity and the glottis **free discharges through a finite system of phonemes specific to each language, by increasing**

their frequency, by accumulating or repeating them, and thus determining the choice of morphemes ... In so doing, the rejection that invests the oral cavity awakens in and through it the "libidinal", "unifying", "positive" drive which characterises, at the earliest stages, this same cavity ... Through the new phonematic and rhythmic network it produces, rejection becomes a source of "aesthetic" pleasure. Thus, without leaving the line of meaning, it cuts up and reorganises that line by imprinting on it the path of drives through the body.

Rejection therefore ... *re*constitutes real objects, "creates" new ones, reinvents the real, and re-symbolises it. Although in so doing rejection recalls the schizoid regressive process, it is more important to note that rejection positivises that process, affirming it by introducing the process into the signifying sphere: the latter thus finds itself separate, divided, put in process ... This symbolisation of rejection is the place of an untenable contradiction which only a limited number of subjects can reach ... this motorial discharge and corporeal spasm are invested in the sign — in language — which is itself already divided, reintroducing and unfolding within it the very mechanics by which the separation between words and things is produced. rejection thus unfolds, dismantles, and readjusts both the *vocal* register ... and the *logical* register (Kristeva 1984, 154–155).

Whether the manual or cerebral spasms of writing can indeed reproduce this process of the vocal system, whereby a semiotic eruption does not destroy, but rather readjusts the thetic position; whether such spasms can produce a logical structure and not just **"aesthetic" pleasure** — these are questions ranging far beyond the reach of this essay. But if we follow this line of thought, and neglect its grip on reality, then the mimetic transcription of metamathematical concepts into arithmetic and formal ones, the persevering enumeration of formulas, and the spastic repetition inside $S(z_p, z_p)$ would be articulated as energetic eruptions of rejection that mobilise Gödel's reformulation of the mathematico-logical field without at all destroying it. We may then view formalists as those who were lost in the rumbling of semiotic motility and succumbed to schizophrenia, whereas

Gödel would become the person who entered the semiotic *Pardes* and emerged with a brand new thetic articulation of self-reference and truth.[17]

Kristeva's thesis is entangled with a Marxist agenda, and her main concern is the relevance of the constructive rejection that she articulates to revolutionary practice. She marks several alternatives. The first is to shatter **conceptual unity into rhythms, logical distortions (Lautréamont), paragrams, and syntactic inventions (Mallarmé), all of which register, within the signifier, the passage beyond its boundary. In these texts it is no longer a question of mere anxiety, but of separation, which is so dangerous for the subject's unity that, as Artaud's text testifies, signifying unity itself vanishes into glossolalia** (Kristeva 1984, 186).

The second alternative, that which is most applicable to Gödel's text, is the subjection of rejection to the narcissistic subject and to the subject of knowledge. **The narcissistic moment tends to attach the process of rejection to the unity of the ego, thus preventing rejection's destructive and innovative vigour from going beyond the enclosure of subjectivity and opening up towards a revolutionary ideology capable of transforming the social machine ... knowledge, to establish itself, will proceed through a supplementary reversal of meaning, by repressing meaning's heterogeneity and by ordering it into concepts or structures based on the divided unity of its subject: *the subject of science* or *theory*** (Kristeva 1984, 186–188). After all, Gödel, while affecting some changes in the rules of the game, never hoped to (and never did) revolutionise logic, not to mention society.

[17]The fact that the relevant psychological biographies indicate the exact opposite is rather ironic. Gödel suffered mental problems, was hospitalised, and eventually died of paranoia (he refused to eat for fear of being poisoned). Georg Cantor is another person whose mathematical views and mental state reflect those of Gödel. The most prominent finitist-formalists, however, Hilbert and von Neumann, seem to have conducted healthy, or at least normal, mental lives. Of course, Hilbert and von Neumann were much more pragmatic with respect to mathematics than the astute position of formalism would suggest, and both Gödel and Cantor were obsessed with a sort of realism that would make it difficult to accept the unstable split of double articulation that, according to psychoanalytic theory, is important for a healthy existence. But since we are discussing the mythical language of pre-crisis mathematics, I impose mythical versions of the relevant characters as well.

If anything, he aimed to strengthen the hold over mathematics of unifying views (realism and present intuition as sources of knowledge).

But Kristeva also believes in a third alternative. An alternative whereby **working with language** could provide rejection with a **free reign to the violence of its struggles — not to founder under those blows, but instead to carry them into the clash of socio-historical contradictions** ... **Constantly keeping the signifying course open to material rejection; preventing the total sublimation of rejection and its repression by introducing it even in the signifying texture [*tissu*] and its chromatic, musical, and paragrammatic differences; and thus unfolding the gamut of pleasure in order to make heterogeneity speak: this constitutes a productive contradiction**. As long as this entire process is kept within the confines of the subject, the capitalist machine can tolerate it. But combining **heterogeneous contradiction, whose mechanism the text possesses, with revolutionary critique of the established social order (relations of production – relations of reproduction): this is precisely what the dominant ideology and its various mechanisms of liberalism, oppression, and defence find intolerable. It is also what is most difficult. In other words, the moment of the semantic and ideological binding of drive rejection should be a binding in and through an analytical-and-revolutionary discourse, removing the subject from signifying experience in order to situate him within the revolutionary changes in social relations and close to their various protagonists** (Kristeva 1984, 189–191).

It seems, today, incredibly naïve to believe that linguistic bursts of drives are the fragile element that is missing from revolutionary practice. But actually, today, any discourse that includes the term **revolution** seems incredibly naïve. A dim and questionable hope however still can be traced: considering the privileged position of mathematics in contemporary intellectual order, could the resurfacing of paradoxical, (non)sensical and genotextual processes, which facilitate semiosis in mathematical texts, contribute towards loosening the grasp of logocentrism, along with its entire political agenda, on contemporary thought?

Conclusion

Everything I've written so far is related to what may be viewed as a universalisable dogma: the sign is materially constituted by bodies, but iterable outside any given context. This universalisability of the dogma, however, is never to be taken for granted. What's interesting about this dogma is not the supposed fact that it is universally true. What might be interesting about it is to show specifically *in what way* it is true for various different texts. It is the singular and contingent manners of forcing different texts to interact through this dogma that might be interesting to explore. Neglecting such particular research is neglecting the specific relations between signs and authorities as they are deployed in the various circumstances of our lives.

I could, of course, give up particular analyses, and try to assess the generalisability of my claims from Gödel's text to other mathematical texts. Doing that would go along the following lines. First, I would show how semantic verisimulation works across a myriad of mathematical texts. Every mathematical concept that is at least doubly determined (and that might cover practically all mathematical objects) would do: geometric objects, which are determined both through geometric axiomatisations and analytic models; numbers, which can be represented geometrically, model-theoretically, or algebraically; predicates, which may be viewed intentionally or extensionally; random variables, which are conceived as throws of dice and at the same time as plain old deterministic distribution functions. In Each of these cases two (or more) things are subsumed under a single sign — all that even before we bring up the relation between mathematical formalisms and whatever they are supposed to model.

Then I would go on to syntactic verisimulation. The authority of logical syntax is felt everywhere in modern mathematics (without necessarily dominating the mathematician's 'everyday' practice). The emphasis on rigid and rigorous rules is no doubt a characteristic of a vast plethora of mathematical languages. And it shouldn't be hard to show how, when a sign is governed by the authority of a repetitive syntax rather than by some attempt to hold on to an essential meaning, this sign is also subject to **dangerous shifts of meaning.**[1]

Next, I could claim that the above **dangerous shifts of meaning** open the way to dissemination and différance. This would require showing how there is always a potential for reiterating syntactic rules in a way that will carry the sign to unexpected semiotic positions, and that will drive it away from any reference it might be supposed to hold on to. The genealogy of a basic geometric notion such as 'line' or a basic algebraic notion such as 'number' would surely reveal a garden of forking paths, which could not be anticipated from the onset, and which is reflected not only historically, but even in the confines of clearly demarcated 'single' texts.

At that point, my analysis of variables, constants and substitution, and the corresponding analysis of equality from chapter 2, are already general enough to cover large tracts of mathematical reasoning that exceed Gödel's text or formal logic. It will then be practically an exercise to extract from all of the above some operative paradoxical elements reflecting the position of infinity, truth and self reference in chapter 3.

In graph theory such paradoxical element would be the node, which is a not yet distributed singular point, at the same time an abstract combinatorial placeless mark and the element determining relative position in a configuration. In probability we have the uniformly distributed point on a segment, which, formally speaking, is not quite a point, but is already rich enough to span entire probabilistic theories. In geometry we have the well known tensions between points as constituting lines and points as object on lines. In set theory we have the empty set, which can constitute the entire universe of sets by embedding itself in ever proliferating arrays of

[1] A complementary tradition of deconstructive research has been dealing with **dangerous shifts of meaning** arising from attempts to hold on to essential meanings, and might be relevant for some less syntactically grounded mathematical discourses.

curly brackets. And in arithmetic we have the constitutive *iterand*, which must be left undefined, but is already nonsensical enough to yield the non a-priori part of mathematics (according to Poincaré), as well as Zeno's and the Skolem-Löwenheim paradoxes. Constituted around such points of reference, the emerging mathematical languages would easily be coaxed to yield concrete expressions of Deleuze's four paradoxes of sense.

All I wrote above is an outline of a programme. This outline alone is obviously nothing but unsubstantiated speculation. But I don't think it would be too hard to follow such a project through. Nevertheless, all this is precisely what I do *not* want to do. Imposing what I've extracted from Gödel via French post-structuralists on the entire field of mathematics would precisely be succumbing to the structuralist fallacy. Even worse; doing that would risk arresting thought. And what I would like to do is keep thought moving. Therefore I will not try to generalise the structures and motions I've explicated in this essay. I will try, instead, to explore further interactions between philosophies and mathematics (e.g. Wagner 2009a,b, forthcoming).

These explorations and those to follow are not about generalising my observations concerning Gödel's text to other mathematical corpuses. That would be to slow thinking down. These explorations generalise a way of thinking that insists on finding in mathematical texts irreducibly unstable cores. These experiments try to respect the peculiarities and contingencies of the analysed texts by confronting them with analyses that turn out productive and challenging on these texts' terms, not on the terms of my former negotiations with Gödel's proof.

As long as my experiments serve the purpose of attracting attention to the problems of authority over meaning and of responsibility for its production in mathematical (and other) texts, I believe there's a point to pursuing them. When the horizons of critical judgement are no longer challenged by these experiments, then, perhaps, the time would come to summarise, universalise, and move on.

Appendix/Overture: Gödel's Myth, Reverse Bricolage

Here we shall pretend to reconstruct a myth, the structural analysis of which is Gödel's proof. The imminent failure is telling of some disparities relating narrative and mathematical texts.

1. Mathematics as the structure of myth

It is commonplace to say that mathematics is a language, and my analysis in this essay will indeed assume as much. But some care is required when taking seriously the statement that mathematics is a language. That mathematics is a language entails a question, which is unique and awkward, in that it rarely requires being asked of other languages: *whose* language is mathematics? Thus wrote Galileo: **Philosophy is written in this grand book — the universe — which stands continuously open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these one is wandering about in a dark labyrinth** (Galileo 1960, 183–184). In this statement mathematics is not a

language that represents the universe, but a language in which the universe is written. It has to be discerned from within the universe. It underlies and structures the universe. It is the language of the universe.

This point of view, which will not be the point of view of the next chapters, is far from extinct today. I thought it necessary, therefore, to begin by reflecting on mathematics as an underlying structure, or as coding something else. Mathematics might very well underlie and structure the universe, or, more modestly articulated, if not the actual universe, at least its reflection in philosophy and rational thought; but acknowledging, on this side of the linguistic turn, that our access to philosophy, thought or the universe depends on language, we are left with the hypothesis that mathematics should underlie the expressions of philosophy, thought and the universe: texts and practices.

To seriously validate or refute the hypothesis that mathematics is an underlying structure would require the confrontation of mathematics with an endless body of texts and practices, which might be structured by it. I do not venture to even embark on such a project. In this chapter I will make only a single modest experiment. I will take one piece of mathematics, and inquire whether it can be regarded as the underlying structure of a very specific kind of textualised thought: mythical narrative. Can Gödel's proof be regarded as the structure underlying a mythical text?

This experiment cannot prove or disprove the viability of viewing mathematics as an underlying structure of anything in general. It will, however, be rather telling in the context of the mathematics/language relation. It will begin to suggest how much we must distort our practices in order to place mathematics behind a non-mathematical text. It will also point out how specific mathematics is, viewed as it will be in the next chapters: not as a background language in the sense posed by Galileo, but as that which is conveyed by a language, laid out in a text, and operated by readers.

My choice to confront mathematics with myth, rather than with any other kind of text, is a contingent choice, which is justified by various pragmatic arguments. First, if mathematics is the language of as exhaustive a book as Galileo's universe, then myth, with its overarching unity spread across distributed authors and story-tellers, is the best literary genre to take

the place of this universal book. Second, this essay offers a post-structural reading of Gödel's proof, and the intricate relations between structuralism and post-structuralism justify starting with a structural analysis as preliminary. As the most highly evolved mechanism for structurally analysing texts is elaborated in Lévi-Strauss' four volumes of his *Introduction to a Science of Mythology*, myth seems to be a good place to start.

We note that there are at least two ways in which Lévi-Strauss' method is supportive of our experiment. First, Lévi-Strauss can be read as allowing for treating structure and meaning rather pragmatically, as interpretive interventions rather than as facts to be simply extracted from data. For Lévi-Strauss **The real question is not whether our endeavour to understand involves a gain or a loss of meaning, but whether the meaning we preserve is of more value than that we have been judicious enough to relinquish** (Lévi-Strauss 1966, 253). He also explains that **the unity of the myth is never more than tendential and projective and cannot reflect a state or a particular moment of the myth. It is a phenomenon of the imagination resulting from the attempt at interpretation; and its function is to endow the myth with synthetic form and to prevent its disintegration into a confusion of opposites** (Lévi-Strauss 1969, 5).[1]

The textual experiment we are about to undertake relies upon these positions. **Meaning**, it says, is an object of **value**, entangled in an economy of **preserv**ation and **relinquish**ment. Unity is **imagin**ed for the purpose

[1]It would be misleading not to counterbalance these two quotations by two other, which reflect the stance of structuralism seeking to uncover structure as underlying the empirico-intelligible interface that must already be there, in disregard to its **value**, simply because it is there: **since, my ambition being to discover the conditions in which systems of truths become mutually convertible and therefore simultaneously acceptable to several different subjects, the pattern of those conditions takes on the character of an autonomous object, independent of any subject** (Lévi-Strauss 1969, 11). Paul Ricoeur called this aspect of Lévi-Strauss' work **Kantism without a transcendental subject**. To see how it integrates with the quotations above, consider that **the methodological rules** that the researcher **will have to evolve in order to translate these** foreign mythical **systems in terms of his own system and vice versa, will reveal a pattern of basic and universal laws**... The mythological code **has neither been invented nor brought in from without. It is inherent in mythology itself, where we simply discover its presence** (Lévi-Strauss 1969, 11–12).

of **synthe**sis. And as such, we choose to bracket the supposed **discover**y and **reveal**ed **presence** of **basic and universal laws**. From this point of view the extraction of meaning and unity from myth is a rather open-ended practice, which is to be judged instrumentally.

Second, Lévi-Strauss' structuring practices are as close as one gets to mathematics in the structural realm. Indeed, Lévi-Strauss warns us that **the occasional use of apparently logico-mathematical symbols** in his book **should not be taken too seriously. There is only a superficial resemblance between my formulas and the equations of the mathematician** (Lévi-Strauss 1969, 30). But in this science-fiction chapter we will pretend that the structural analysis of myth has in fact reached the postulated **future**, where it will be **possible to subject myth to a genuine logico-mathematical analysis** (Lévi-Strauss 1969, 31).

These circumstances give our experiment fair grounds for productivity. They allow us to ask: *what is the value of preserving or relinquishing Gödel's proof, not as a meaningful text, but as the meaning of something else (a mythical narrative)?* More precisely, we ask whether we should assign to Gödel's proof the synthesising function of structural unity inherent in something else (a mythical narrative), or whether it is of more value to consider it as meaning and structure laid out in text, as, to a certain extent, we eventually will.

Our experiment will consist of pretending that there is a lost myth, or perhaps a collection of lost myths, which Gödel analysed structurally in two similar ways (his 1931 and 1934 texts). We will pretend that all we have left from the myth is Gödel's analysis. Our aim is to recover the underlying myth. To do so requires the production of a narrative supposedly structured by the proof, which looks like a possible myth.

The myth I pretend to recover is a creation myth. It recounts how certain creatures came to be created, named, and born, and how the incestuous rules governing their birth ended up undoing the order of the created world. This attempt of excavating a myth from underneath Gödel's proof will fail an instructive and glorious (in the Humpty-Dumpty sense) failure.

The next section will follow the ingredients and argument of the proof, and in parallel reconstruct the myth, which we pretend that the proof structures. As the proof grows more intricate, so will the myth grow more

involved. The third section will analyse the failure of this odd exercise, and discuss what we can learn from this failure concerning mathematical texts.

2. The myth

2.1. The creators

We might expect a creation myth to articulate a creator position, but suppress the narrator position. Gödel's text, however, confounds the two positions. On the one hand it makes such statements as: **We now proceed to carry out with full precision** the analysis **sketched above** (1931, 151), assuming the narrator position, talking about the myth. On the other hand Gödel's text includes such comments as: **We now assign natural numbers to the primitive signs** (1931, 157), performing analytic manipulations, which structurally reflect the creative mythical actions of creators in the myth (actions that we shall later retrace to mythical 'naming').

In both cases **We** is at once singular and plural. In the first case, it is the scientific We, which imposes on the reader the actions of the writer by referring them to an abstract communal entity. In the second case it is a creating power, whose distinguished position is manifested by a plural mark (the royal 'We', the Hebrew plural divine name 'Elohim', as well as the Christian God who is three who are one).

Clearly, this confounding is not accidental. It reflects Gödel's articulation of the structural creator position in the myth as homologous to the contemporary structural position of the scientific We. The creating power in the myth below, like the scientific We, should therefore be conceived as a pervasive communal spirit, which is not subject to the whims of any individual component, and yet exists only insofar as it succeeds in possessing many of these individuals. Like the scientific We, this form of possession, however, is not an arbitrary external compulsion. It has the curious feature of simultaneously imposing an external edict on every individual in the community, and but conditioning the validity of this edict on the individuals' communal reaffirmation.

We conclude that the myth underlying Gödel's structural analysis was not recounted from the point of view of subjected creatures (as in the

priestly formula of 'God created light, the earth, and eventually us humans', where creation requires no scrutiny), but from the point of view of the creating powers themselves, who form a critical discursive community, perhaps somewhat like the Greek Olympus. As we shall see, this structural position implies, like in the case of the Olympic gods, that the creating powers are not an absolute beginning, but rather a link in a complex hierarchy of creation, subject to an open-ended genetic chain that stretches beyond any individual creator.

2.2. Primary articulation: elements of the world

The creating power (the above **We**) is to set the elements of which the world, structurally transformed by Gödel into a *Formal System*, consists. These elements are structurally described as *Primitive Signs* in a logical formal system: an infinite pool of *variables*, which are further divided into *types*, and a finite collection of *constants* — logical connectives, numerals, basic predicates and functions, together with punctuation marks such as parentheses.

Figure 1: List of logical constants and variables in the formal system that we will use. I diverge somewhat from Gödel's notation in order to make things a little more tractable.

Logical Constants:

- Connectives: $\neg$ (not), $\wedge$ (and), $\vee$ (or), $\rightarrow$ (implies)
- Quantifiers: $\forall$ (for all), $\exists$ (there exists)
- Parentheses: (,)
- Numeral: 0
- Function: +1

Logical Variables:

- Propositional variables: $p_1, p_2, p_3 \cdots$ (standing for propositions)
- Numerical variables: $x_1, x_2, x_3 \cdots$ (standing for numerals)
- Function variables: $f_1, f_2, f_3 \cdots$ (standing for functions)

In structural terms, these are the elements of a primary articulation: meaningless components, explicitly referred to as **undefined terms** (1934, 346), not unlike phonemes. One should refrain from taking this primary articulation too seriously. As Lévi-Strauss admits, **As happens in the case of an optical microscope, which is incapable of revealing the ultimate structure of matter to the observer, we can only choose between various degrees of enlargement: each one reveals a level of organisation which has no more than a relative truth and, while it lasts, excludes the perception of other levels** (Lévi-Strauss 1969, 3).

The list of elements, therefore, corresponds not to some absolute basic elements of the underlying myth, but to the structural necessity to mark a point of departure. This explains why the two analyses offered by Gödel (the 1931 and the 1934 texts) diverge as to the exact identification of the elements of the formal system that structures the underlying mythical world.[2]

Since this level of articulation is at least partly arbitrary, it alone does not enable us to retrace the underlying elements of the mythical world. We shall have to settle for assuming that the created mythical world could be broken down, although perhaps not in a unique and unequivocal way, into a set of distinct elements, which Gödel managed to retrieve from the myth.

2.3. Secondary articulation: creatures and names

The next step in Gödel's structural analysis is to define the laws of the secondary articulation, that which collects elementary signs to form meaningful formulas. These are fairly standard, but, since we have two versions of a primary articulation, we have some divergence concerning the exact rules of the second articulation as well. Since I could not retrace the exact mythical details underlying each element in each version, there is no point in presenting Gödel's elaborate formal system in full.

Mythically, the second articulation is a structural generalisation about the entities that the created world could host. These are not yet necessar-

[2]One analysis allows for infinitely many types of variables, while the other for only three; one allows for 7 constants, while the other names 13; one decomposes the infinite list of variables into finite combinations of finitely many numerals, while the other is content to invoke an infinity of elementary variables.

ily fully fledged living mythical creatures, and, obviously, not all entities allowed by the mythical rules of combination (as reconstructed in Gödel's analysis) will have a role in the myth itself.

It is crucial to note that Gödel's structural analysis allows logical formulas containing free variables that can eventually be substituted by non-variable expressions. Some mythical entities, therefore, have the structural role of moulds that can be substituted into. It is also important to note that some formulas can be combined to form larger formulas. Some mythical entities, therefore, should be regarded as limbs and organs that can be put together.

Figure 2: Combinations of signs that follow some carefully specified syntactic rules (which we omit) are called formulas.

The following are examples of formulas.

- $(0+1)+1$ (the number 2)
- $x+1$ (a number form with x as free variable)
- $x=0$ (a proposition containing x as free variable)
- $\forall x(x=0)$ (the (false) proposition 'all numbers equal zero' containing x as bound variable)

The following are examples of combinations of signs, which do not meet the syntactic criteria for formulas

- $((x\neg$
- $\forall x$
- $(x=0) \rightarrow$

Within this complex zoology, there is a marked division of entities into two main kinds. The first kind is structurally interpreted as *numbers* and as *number-forms* (the latter are expressions containing variables, which may be substituted for so as to form numbers; e.g. $x+1$, which is not yet a number, but will become a number if we substitute, say, 4 for x). The second kind is structured as logico-arithmetic *propositions*, namely claims concerning variables or numbers. Both these kinds of entities are generated from the same basic elements of the primary articulation following carefully

specified rules. Entities of either kind may be composed with additional elements to create new entities of either kind.

What is the mythical division that gave rise to the number/proposition structural dichotomy? The reconstruction has at this stage very little to go on. Among the huge plethora of imaginable mythical objects one must find a fundamental ontology that respects the structure offered by Gödel's text. At this stage everything is still highly underdetermined. It is only the structural analysis in its entirety, which can correspond to an underlying myth. The isolation of partial constituents of the structural analysis will necessarily leave too many degrees of freedom. The reader is therefore asked to indulge my suggestions, and judge them only when the account is complete (the proof of the pudding is in the eating).

My suggestions are that the first structural entity, numbers and number-forms, corresponds to names, parts of names, and moulds that may become names. The second structural entity, propositions (with or without free variables), shall correspond to bodies, parts of bodies, and moulds that may become bodies of creatures inhabiting the mythical world. Before we go into the relations between bodies and names, we shall have to be a little more specific concerning the mechanisms through which the basic elements can be composed to form bodies and names.

Three forms of creation (composing elements to form bodies and names) must be explicitly described, as without them the myth cannot be elaborated. The first is structurally articulated as adding the connective $\neg$ ('not') to a propositional formula. The underlying mythical relation is reconstructed as that of *nemesis*. For each body, the powers **We** can create a nemesis. A body and its nemesis are in a relation of strict exclusion. They are not supposed to be both born into the created world (more on this below). The question of which should be born — a creature or its nemesis — is a main concern addressed by the myth.

The second and third forms of creation are closely related. One is structurally articulated as the substitution of a constant for a free variable (for instance, turning the formula $x > 0$ into $4 > 0$). The other is structurally articulated as binding a free variable (for instance, turning the formula $x > 0$ into $\forall x(x > 0)$). Free variables, in Gödel's system, are not supposed to remain free indefinitely. We can therefore infer that free

variables are the structural representation of incompleteness in bodies and names, cavities that must eventually be filled.

These formal manoeuvres suggest that some entities created by **We**, our myth shall recount, are incomplete. They are only moulds. They contain cavities (free variables) that have to be replaced if the incomplete body — which, as we shall see, can be viewed as a sort of *Golem* — is to be animated. These cavities seem to include the mouths and vaginas of mythical creatures. The substitution of a number for a free variable is the structural representation of feeding a name into a Golem's mouth cavity, thus producing an animated creature, following the example of Jewish myth.

As for binding free variables — there the difficulty is more pronounced. In one sense, a bound variable is replaced by all possible values, and we can think of feeding the Golem with all possible names. An alternative image is gagging (binding) the Golem's mouth cavity, preventing it from being fed a name. Indeed, a bound variable, on the one hand, is not substituted, but on the other, can no longer be substituted. But for reasons which will become apparent later, I suggest a third (less than perfect) solution. Binding a variable will be taken as the structural formulation of replacing a vagina-cavity by a phallus, a sort of symbolic castration.[3] The procreative associations will prove useful for retracing the myth.

Finally, if a free variable structuring the mythical role of a vagina-cavity is substituted rather than bound, the myth would have to recount how a name was inserted into the vagina. This operation will be mythically identified as impregnation.

So far we have a creating power **We**, the basic elements (primitive signs) of which a world (formal system) is created, the rules that allow composing the elements into parts and moulds of Golems and into parts and moulds of names, and finally some distinguished forms of creation: a nemesis for each creature (adding a $\neg$ prefix), feeding a name to a Golem (substituting a number for a free variable), replacing the vagina of a Golem by a phallus (binding a free variable), and impregnating a Golem by placing a name into its vagina (substitution again).

[3]This might have some obscure relation to Philo of Alexandria's interpretation of the Jewish creation myth as consisting of creating first a heavenly man, then an earthly man, and only subsequently gendered man and woman.

It is important to understand that a fed, impregnated, or phallus-endowed Golem is not the same creature as the original Golem. Indeed, any manipulation of a formula turns it into a new formula, and therefore any manipulation of a mythical individual turns it into a new individual entity, which will later be endowed with a new individual name.

2.4. A transformation within the secondary articulation: from creatures to names

In Gödel's structural analysis, each primitive sign is assigned a numerical value; as a result logical formulas, which are sequences of primitive signs, correspond to sequences of numbers. These number sequences are compounded via explicitly stated arithmetic operations to produce a single number. The result is that each sequence of signs (and in particular, each formula) is assigned a number. The apparently strange arithmetic operation governing the assignment (see the Figure below) is chosen so as to guarantee that no two sequences share the same number, and no two numbers share the same sequence of signs.

This strange structural description seems to recapture a mythical system of expressive naming. The name of a creature is calculated from its composition. Each of the world's elements has a name, and the name of a creature is produced by combining these names in a carefully prescribed manner. The system of naming is so constructed, that no two creatures are ever endowed with the same name.

We must introduce a certain duality into our reconstructed myth in order to account for the logical division between *arithmetic* numbers, which appear when talking *about* the formal system (such as the arithmetic number 4), and representations of numbers by numerals *in* the formal system (such as the string $((((0) + 1) + 1) + 1) + 1$, which represents 4). Naming, therefore, cannot occur in the world of the created (the formal system). A creature's true name exists in the thought of the creating powers **We** (the realm of arithmetic). But **We** can represent (or write) names by objects that can be introduced into the world (number-formulas in the formal system), and these name-representing-objects (hieroglyphs?) can impregnate or be fed into Golems. The true name is not a created object among others,

but it can be represented by a created object. This mythical element might have something to do with Walter Benjamin's myth in *On Language as Such and the Language of Man* (Benjamin 1966), which builds on a long tradition of reflection on divine vs. human languages.

Figure 3: Transforming logical formulas into numbers.

Each primitive sign is assigned a number.

- $\forall$ is assigned the number 1
- (is assigned the number 2
-) is assigned the number 3
- x is assigned the number 4
- $=$ is assigned the number 5
- 0 is assigned the number 6

etc.

Each formula then corresponds to a sequence of numbers. The formula

$$\forall x(x = 0)$$

corresponds term-by-term to the sequence

$$1, 4, 2, 4, 5, 6, 3.$$

This sequence is converted into a product of prime powers. The bases are the prime numbers, and the exponents are the elements of the corresponding sequence:

$$2^1 \times 3^4 \times 5^2 \times 7^4 \times 11^5 \times 13^6 \times 17^3$$

$$= 37,137,912,361,784,015,131,350$$

The resulting number is not an element of the formal system, because it is not a sequence of primitive symbols of the formal system. This number, however, can be represented in the system as a long chain of +1's: $(\ldots((0+1)+1)\ldots+1)+1$

The procedure of naming (which, again, is somewhat different in Gödel's two structural accounts) allows for a highly intricate cosmology. Manipulations of the composition of creatures correspond to manipulations of their

names. At the same time, whatever **We** do with these names can be reflected by representations of names in the created world. A link is formed between creatures, their true names, and the representations of these names in the created world.

2.5. Tertiary articulation: birth

So far Gödel's structural analysis made no reference to semantics. We had syntactic rules governing the composition of primitive signs into logical formulas, but ignored the question of whether these formulas are true or false. The next element in the structural analysis is the notion of proof, which is a syntactic precursor of a semantic division.

Gödel's structural analysis lays down explicit syntactic rules that determine which sequences of logical formulas are to be accepted as proofs, and constitutes a division between provable and non-provable formulas. Provable formulas are simply those that can appear at the bottom line of a proof.

What is the mythical element structured in this way? We view proofs as the generation of formulas from other formulas, and hypothesise that this structure emerged from a mythical concept of begetting. Many formulas can be formed, which might never be proved. Equivalently, not all creatures we have discussed so far will be born into the world. The introduction of creatures into the world of the living is selective, and subject to an explicit articulation.

Again, the list of birth/proof rules is tedious. It begins with axioms, which structurally determine the first creatures, unborn, introduced by **We** into the world of the living. It then specifies rules of inference, which structure laws of birth, specifying which lineages can give birth to which creatures. This technical and tedious account may echo the long and tedious biblical genealogies of the books of Genesis and Chronicles.

Note that in the reconstructed mythical world, as in many other mythical worlds, a creature does not necessarily require two parents. Some forms of birth involve a single parent, while others involve more. In our mythical world a birth is ascribed to an entire lineage (a proof), starting from the first creatures introduced into the world (axioms). The structural analysis

— and therefore, we must assume, the myth too — generalises the rules assigning numbers (names) to formulas (creatures) into rules assigning numbers (names) to sequences of formulas (lineages). Not only individuals, but entire dynasties as well, are assigned expressive mythical names.

For our purposes we need to single out one important formal rule of inference. This is the rule that allows to derive from the formula $\forall x P(x)$ (for every x, property P holds of x) the conclusion $P(a)$ (property P holds of a), where P is any predicate, and a is any number-formula. This rule allows to replace the binding of a variable by a substitution. In mythical terms, this rule allows a Golem, whose cavity is replaced by a phallus (the variable in its formula is bound), to give birth to any Golem of the exact same composition, whose cavity is impregnated by any name (the variable in the formula is substituted by some number-formula). In our myth, which seems to draw here upon the myth of symbolic castration, the phallus is the master-father of all names.

Gödel's structural analysis explicitly recommends to forget the issue of proofs (the component of the myth related to begetting) while introducing the issue of numbering formulas (the component of the myth relating to naming). It could be that the two myths derive from two different sources. It is clear, however, that the myth, which we are about to unfold, is the product of combining these two mythical sources. It seems that Gödel's analysis demonstrates how two different mythical structures interact with each other to yield a crisis, and how the mythical *bricoleurs* manage to bring the crisis into a safe resolution, which reconciles it with the demands of the *Savage Mind*.

2.6. Refining the tertiary articulation: birth oracles

We seem to have believed that the rules governing begetting guaranteed a harmony in the world of the born. First, **We** believed that if a creature is born, its nemesis cannot be born. This is structurally presented as the law of contradiction: a formula and its negation should not both be provable. **We** also seem to have believed that if a person is not born, then its nemesis must eventually be born. This is structurally reflected by the law of excluded middle: for any formula, either it or its negation should be provable. But

We's belief was clouded by the shadow of a doubt.

We indeed made precise rules to determine which lineages can give birth to which creatures. But if **We** were to determine whether a certain creature could be born, **We** would have to check whether it could be born of each and every possible lineage — an infinite task, which even **We** could not perform.

Concerned about foreseeing the vicissitudes of procreation, **We** set out to create oracles. A heavily constrained and intricate ritual resulted in the creation of a Golem called *Bew*[4] (structurally represented by a certain intricate predicate marked 'Bew'). Bew had two cavities, a vagina and a mouth. To determine whether a certain creature could be born to a certain lineage of ancestors (whether a certain sequence of formulas proves a certain conclusion), **We** would impregnate Bew with the name of the lineage, and feed Bew with the name of the creature (this corresponds to substituting the variables in the 'Bew' predicate). If a thus impregnated-and-fed-Bew was subsequently born into the world (the 'Bew' predicate, after substitution, is provable), then the said creature could indeed be born to the said lineage (the sequence of formulas indeed proves the conclusion). If the impregnated-and-fed-Bew's nemesis was born (the negation of the 'Bew' predicate after substitution was provable), then the creature could not be born of the lineage (the sequence of formulas does not prove the conclusion).

One birth — that of impregnated-and-fed-Bew or of its nemesis — was to foretell another birth — that of a certain creature to a certain lineage. What made Bew a useful oracle was that, since Bew was created via a specially constrained ritual, **We** could always contemplate their way into discovering whether the impregnated-and-fed-Bew — or its nemesis — were to be born (one can always verify whether 'Bew' predicates, after any substitution, are provable or refutable). This feature of the myth is structurally captured by the logical feature of *recursive functions*,[5] which allows to transform some problems of provability, including the provability of Bew predicates, into mechanical arithmetic computation. Impregnated-and-fed-Bew serves as an oracle, because **We** do not really have to wait

[4]Since the structural analysis provides no hints concerning the names of the relevant characters, I will make up names from the notation in Gödel's text.

[5]In today's terms, *primitive recursive*.

until it, or its nemesis, is born (until someone discovers a proof). **We** can foresee its birth in **We**'s minds (through some computational algorithm), and subsequently foretell the births of other creatures to given lineages.

This is the most intricate, the most tedious, the most technical part of Gödel's structural analysis. Over 4 pages, including 45 steps, the mythical ritual of the creation of Bew is recast into the structural language of recursive functions. The structural description is so intricate, that every step is not only formally written, but also informally explained. It is in fact so tedious, that Gödel's second structural account (the 1934 text) only summarises the process. Obviously, the structural analysis is based on a mythical incantation, which included an extremely detailed ritual protocol, and which we have no hope of recapturing. This ritual served to guarantee that **We** could tell in advance (compute algorithmically) whether an impregnated-and-fed-Bew was to be born, that is, whether the lineage (sequence of formulas), whose name (number) impregnated (was substituted into one free variable of the predicate) Bew, could give birth to (proves) the creature (formula) whose name (number) was fed into (was substituted into) Bew's mouth (the other free variable in the 'Bew' predicate).

Did the original myth really contain such an incredibly intricate account of the Bew's creation ritual? It is hard to believe that any myth should be so incredibly detailed. Perhaps the difficulty here was in recapturing a simple line of thought by Gödel's structural language. One should bear in mind that for the people who originally recounted the myth, the ritual, albeit described in full detail, was only a metaphysical reality. If the original recounters of the myth did insist on such minute detail, it would have served to convince them that they (or **We**) could, in principle, reproduce the ritual, given enough time and resources. But the fact remains that, living among the real concerns of the real world, having little to do with mythical creatures, they would never have had the motivation to even try — the point of the myth obviously being, as structuralism instructs us, its message or structure, and not its imagined replication.

2.7. A crisis in the code

With the creation of Bew the conclusion of the myth is near. **We** were not content with Bew alone. **We** started remoulding and varying it. The immediate motivation for these manipulations is not quite coded inside the structural analysis, although Gödel does provide a motivational analogy by bringing up Richard's and the liar paradoxes. Perhaps the mythical motivation had to do with the ever fateful quest for absolute knowledge.
Perhaps **We** wanted to know more about the future inhabitants of their world. Perhaps this is one of those myths where an excessive search for knowledge wreaks havoc. On the other hand, perhaps **We**'s motivation was absent from the myth itself, which, we presume, was more intent on the consequences. Perhaps **We** just followed Lévi-Strauss' terms and relations inversion formula of his structurally constitutive **kind of permutation group** (Lévi-Strauss 1976, 228).

At any rate, **We**'s experiments resulted in the creation of *Qew*,[6] a modified nemesis of Bew, which had the following queer oracle capacity: impregnated with the name of a lineage, and fed the name of a Golem, Qew disclosed whether the lineage could give birth, not to the named Golem, but to the Golem-fed-its-own-name (this self-feeding structurally translates as a formula, whose free variable is substituted by the number that designates that very formula). If that birth was possible, impregnated-and-fed-Qew's

[6]For those who wish to keep track of the relation between this section and Gödel's proof, here are the relevant definitions:

- When x is a number, z_x is the corresponding numeral in the formal system.
- $Bew(z_x, z_y)$ is a predicate computable in terms of recursive functions, which is provable if and only if the string of sign sequences numbered x proves the formula numbered y.
- $S(z_a, z_b)$ is the numeral representing the formula obtained by substituting z_b into all free occurrences of the variable w in the formula numbered a.
- $Q(v, w) = \neg Bew(v, S(w, w))$.
- z_p is the numeral representing the formula $\forall v Q(v, w)$.
- It follows that $S(z_p, z_p)$ is the numeral corresponding to the formula $\forall v \neg Bew(v, S(z_p, z_p))$.

As a result the numeral $S(z_p, z_p)$ belongs to a formula, which claims of itself that it is unprovable. Gödel's proof can now be concluded as in the introduction.

nemesis would be born. If the birth could *not* take place, impregnated-and-fed-Qew would be born. The birth of Qew, impregnated with a lineage and fed a Golem's name, was tantamount to the following prophecy: this lineage cannot give birth to that Golem-fed-its-own-name.

The Golem Qew was then endowed with a phallus to replace its vagina. According to the laws of begetting, if Qew-endowed-with-a-phallus-and-fed-a-Golem's-name was born, it could then give birth to Qew-impregnated-with-any-lineage-name-and-fed-the-Golem's-name. Such swarm of births, according to the paragraph above, would prophesise: no lineage could give birth to this Golem-fed-its-own-name. In other words: This Golem-fed-its-own-name cannot be born! Qew endowed with a phallus came to be known as *Zp*.

Figure 4: The oracle Qew.

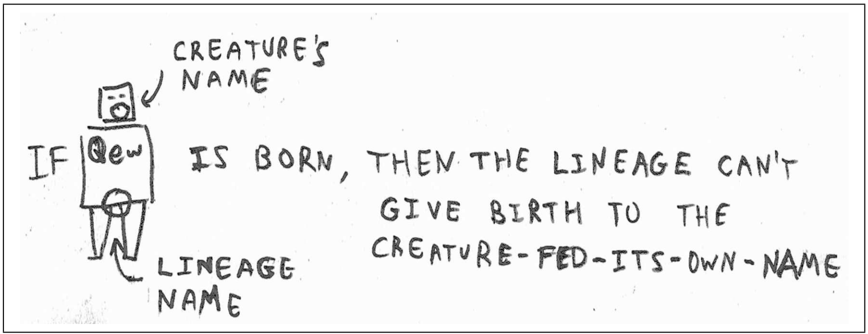
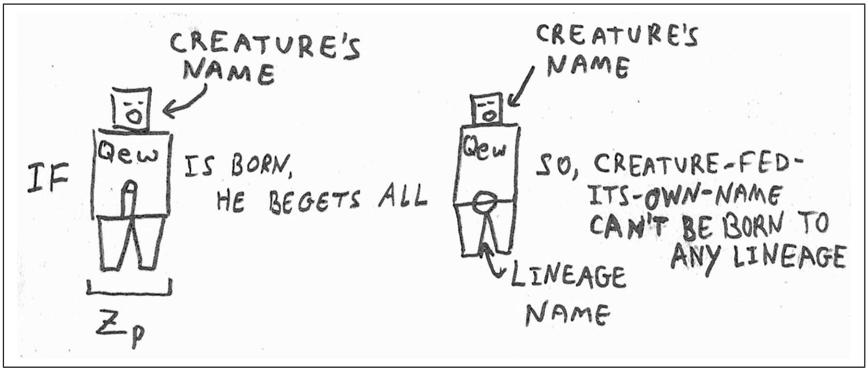
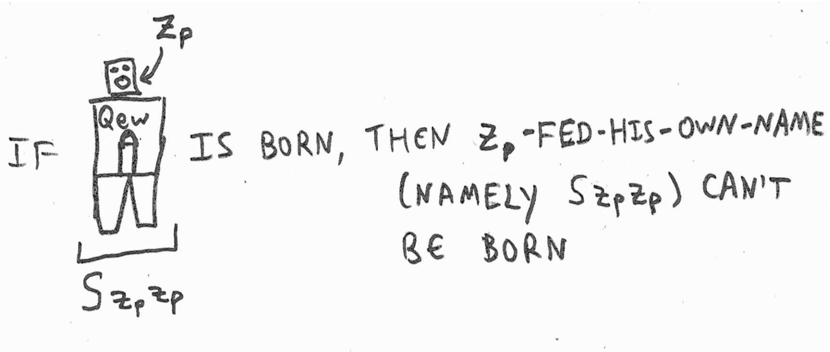


Figure 5: Zp — Qew endowed with a phallus.



Finally Zp was fed its own name. The birth of Zp-fed-its-own-name, which is none other than Qew-endowed-with-a-phallus-and-fed-Zp's-name, would signify (according to the previous paragraph): Zp-fed-its-own-name cannot be born. To put things succinctly, Zp-fed-its-own-name was a creature whose very birth would prophesise that he himself could not be born. This strange contorted and grotesque creature was named *Szpzp*.

Figure 6: Szpzp — Zp fed his own name.



Surely, such a creature as Szpzp cannot be born into a harmonious world. And, since Szpzp will not be born, its nemesis must be born. Now Szpzp prophesied that Szpzp won't be born; therefore the birth of Szpzp's nemesis must prophesise that Szpzp would be born, which would bring, as above, disharmony again![7]

[7]This is some sort of weak form of the semantic argument presented in the analytic introduction. Trying to mythically capture the syntactic argument is even more awkward and unpleasant. Here is how it would go:

The error, however, **We** hoped, was in their own reasoning, in their careless analysis of the situation, and not in the created world itself. Entangled in contemplating the monstrosity in their mind, **We** went into a deep meditative sleep, trying to carefully foretell the destiny of Szpzp.

In **We**'s meditation, **We** dreamt that Szpzp (that is, Zp-fed-its-own-name) was born into the world of the living, fathered by some obscure, unknown, lineage. This birth must have been reflected by the oracle Golem Qew: the nemesis of Qew-impregnated-with-the-obscure-lineage-name-and-fed-the-name-Zp had been born. The days went on, and Szpzp went on to bear his own children. And as Szpzp was but the Golem Qew-endowed-with-a-phallus-and-fed-the-name-Zp, it begot Qew-impregnated-with-the-

We were obviously upset. The world seemed to be bound to disharmony regardless of which was to be born, Szpzp or its nemesis. The hope that for any possible creature, either it or its nemesis would be born — but not both at once — was shattered. The entire created world was at risk of being destroyed by its own dismayed creators...

2.8. Rehabilitating the code

Gödel's structural analysis goes on to explain that the structure discovered is in fact a common feature of many myths, not only the two versions of the myth analysed in the 1931 and 1934 texts. Any creation myth (formal system) based on a world with explicitly verifiable rules of birth (inference rules), a system of names (the natural numbers), a notion of nemesis (negation), a phallus (a universal quantifier), and hypothesised harmony (law of contradiction) must have this structure. Gödel must have been using the large variety of existing formal systems to represent a large collection of variants of the myth he analysed.

Gödel also seems to have found an appendix to the myth, which, I would guess, came from a later source, representing some evolution in the social structure underlying the production of the myth. To his structural analysis he appended remarks suggesting that the notion of proof should be separated from the notion of truth: every formula proved must be true, but not all true formulas can be proved. Provability, in the structural analysis, is relative to a formal system. Truth, on the other hand, exceeds any formal

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obscure-lineage-name-and-fed-the-name-Zp. Into this dream — or rather this nightmare — world, were born both an impregnated-and-fed-Qew and its very own nemesis. Harmony was broken. **We** woke up, and rejected the dream as false.

The following night, **We** dreamt of a world where Szpzp could not be born, and instead Szpzp's nemesis was born. This, again, was prophesied by the Golem Qew. Into this second dream world were born a swarm of Qew's: Every Golem Qew-impregnated-with-any-lineage-name-and-fed-the-name-Zp had been born, testifying that Zp-fed-its-own-name (namely Szpzp) could not be born to any lineage whatsoever. But this swarm of creatures were precisely all the children of Qew-endowed-with-a-phallus-and-fed-the-name-Zp — of Szpzp himself. Into this dream — or rather nightmare — world, were born the nemesis of Szpzp, as well as all of Szpzp's children. This world too was deemed disharmonious. **We** woke up, and rejected that dream as false as well.

Contemplating these dreams **We** faced a grim reality: Neither Szpzp, nor its nemesis could be allowed into the world of the living without breaking its harmony.

system.

This step seems to reflect the following mythical manoeuvre. Neither Szpzp nor its nemesis can be born into the world of the living without upsetting its harmony. However, higher worlds (formal systems) may exist, which look upon the world of **We**. And these higher vantage points allow to verify that Szpzp, while it should not be allowed into the world of **We**'s born creatures (it is not provable in the given formal system), does conform with the harmony of the larger universe (it is true, and also provable in higher formal systems). None of these higher vantage points is perfect. In each such 'higher world' **We** could contemplate disharmonious creatures. But these creatures, it is hoped, could be placed in the larger scheme of things when considered from higher points of view. The structural analysis seems to indicate that all these higher vantage points can be conjured in the thoughts and dreams of the creating power **We**. At the same time, it gives the distinct impression that **We** are not omnipotent, but subject to some unattainable higher cosmic harmony.

A clear formulation of this peculiar structure is not provided in full, and seems to have been deliberately avoided in Gödel's analysis. This evasion may be rooted in Gödel's concern that such further analysis may be criticised as not relying on explicitly documented myths but on Gödel's own views of structure. Another hypothesis is that this complete hierarchy is beyond the expressive power of Gödel's language of structural analysis; alternatively (assuming that Gödel's language of structural analysis was perfectly adequate), the complete myth, which would express this all encompassing hierarchy, never was — perhaps never could be — told. This hypothesised complete myth (the myth of logical truth) reflects the unattainable *telos* of evolving formal systems — or of mythical structures.

3. This is not a myth

Of course, I could insist that the exercise was successful. Not only did I manage to reconstruct a myth underlying Gödel's proof, but this myth is well correlated with some prominent structural motifs. My improvised myth, an anthropological structuralist zealot could argue, is all about how breaking totemic and incestuous taboos (eating one's own name, being

impregnated by one's own ancestors) leads to a breach of social harmony. This would in fact be a major triumph for the science of mythology. It would imply that even a mathematical text is, up to a transformation, some sort of analogue of the Tukuna myth analysed by Lévi-Strauss (1978, Part I), in that it recaptures the structure of improper coupling, or of the Oedipus myth, in that it asserts the relation between riddles (questions without answers, undecidable statements) and incest (Lévi-Strauss 1976, 21–24).

A fundamentalist psychoanalytic structuralist could join in as well. Gödel's proof turns out to affirm the danger of forcing an equivalence (allowing replacement) between the name-of-the-father or the phallus and other proper names, and brings out the hazardous repercussions of failing to set asunder the subject of enunciation and the enunciated subject (in such circumstances as when a creature carries the message that he-himself cannot be born) (cf. Lacan 1978, 139).

If this zealot and that fundamentalist successfully survived the seventies and persist on living to this day, far be it from me to upset their omniscience. But I believe most readers would prefer to reject the story I have pretended to uncover as underlying Gödel's proof, and not only due to the trivial and boring fact that no myth ever actually did underlie the proof. I will devote the rest of this chapter to analyse the various ways in which my myth fails both as a myth and as a structural equivalent of Gödel's proof, and to study what, if anything, this failure can teach us concerning the functioning of a mathematical text.

3.1. This is not a myth

I do not deny (in fact, I strongly embrace) the claim that every mathematical argument is accompanied by some *sort of* narrative. E. M. Forster's[8]

[8]It is not terribly easy to justify my choice of Forster as authority. His analysis is indeed a classic, but is rather far behind contemporary research in narratology. Nevertheless, his definitions and critique provide me with more than enough to go on, and in our context his argument is still convincing even today. Another reason is Forster's special position (both as theoretician and as writer) observing a crucial junction between English and French literary modernism. This position echoes the special position of structuralism as a juncture between analytic reasoning and continental thought. But I must also concede that I choose to quote Forster as homage to his beautiful work.

definitions of story and plot as forms of narrative are enlightening in this respect. **Let us define a plot**, he suggests. **We have defined a story as a narrative of events arranged in their time-sequence. A plot is also a narrative of events, the emphasis falling on causality. "The king died and then the queen died," is a story. "The king died, and then the queen died of grief" is a plot. The time-sequence is preserved, but the sense of causality overshadows it. Or again: "The queen died, no one knew why, until it was discovered that it was through grief at the death of the king." This is a plot with a mystery in it, a form capable of high development. It suspends the time-sequence, it moves as far away from the story as its limitations will allow. Consider the death of the queen. If it is in a story we say 'and then?' If it is in a plot we ask 'why?' That is the fundamental difference between these two aspects of the novel. A plot cannot be told to a gasping audience of cave men or to a tyrannical sultan or to their modern descendant the movie public. They can only be kept awake by 'and then – and then —' they can only supply curiosity. But a plot demands intelligence and memory also** (Forster 1927, 82–83).

We can't deny that a mathematical argument is indeed a sequence of assertions tied together by a chain of **why?**s. Let us put aside for now the question of whether 'assertions' can indeed stand for **events**. A host of theoreticians will endorse such affinity, while quite a few others will protest. But if we replace **event** by 'state of affairs', and accept that even if mathematics is not quite about 'states of affairs', our understanding of it does nevertheless make an at least allegorical use of this term, surely we could bring Forster's definition of plot and mathematical texts close enough together to espouse them to each other on the grounds of the operative **why?**.

But Forster's definition is also useful for stating an objection to the claim that my narrative above can serve as a myth. The cave dwellers' myth revolves around **and then?**. My narrative does indeed attempt to suppress the mathematical **why?**, and linearise it into a sequence of **and then?**s (the question why is such-and-such unprovable is replaced by a sequence of 'and then this was created, and then that was created, and then it was

prophesied that this couldn't be born, and then...'). But this sequence of **and then**s is precisely the kind of sequence that wouldn't keep any cave dweller community awake. It is too hard to follow; the interrelations are too complex and have too little to do with daily lives; the shadows of the mathematical fetish for justification (**why?**) hang over the narrative to the point of paralysing it. The trace of the **why?**, even if no 'because' explicitly appears in my story, is not quite suppressed. Respecting the structure of mathematical justification is (unfortunately?) related to repressing, to an extent, an enthraling **and then?**. A mathematical proof, it seems, may have the making of a plot (**why?**), but lacks the making of a story (**and then?**).

But indeed, the line between **and then?** and **why?** is not all that clear-cut. More than anything, this distinction is an approximate excuse for the evident failure of my impromptu myth to enthral. The question could be raised, therefore, what if a more talented story teller (or better yet, a myth-telling community) were to take my text and process it? Could it then come to function as a myth? Suppose indeed that it did. Would the transformed myth, suppressing the **why?** for **and then?**, still be structurally reducible to Gödel's math?[9] The answer is bound to depend on the motivation of the person by whom this myth is to be reduced, and on their excavation skills. Indeed, the most 'scientifically' oriented critique against the structuralist study of myths is that it lacks a clear articulation of *mythèmes* (mythical elementary units), which makes the analysis rather arbitrary (e.g. Sperber 1973). I believe that only through such arbitrariness could the structure of the proof be observed in such hypothesised myth. But since the suggested myth-reprocessing experiment is likely never to take place, I leave this doubtful train of thought and proceed in another direction.

3.2. This is not even a plot

If we had excavated a plot behind Gödel's text, the question whether this plot did or did not conform to the requirements of the mythical genre could be confined to the margins; such a feat would be a remarkable enough achievement in itself. But while I did acknowledge that every mathemat-

[9] I will shortly turn to discuss whether my own text, as it stands, is in fact so reducible.

ical argument is *accompanied* by some sort of narrative, I do not believe that a plot can exhaust a mathematical argument, or *lie behind* it. Some sorts of plots are often *produced* by mathematicians so as to accompany their understanding of mathematical texts. But to ask for a relation more fundamental than that is to impose. As we shall detail now, my attempt above indicates some typical obstructions.

My myth refers to several levels of articulation, but provides almost no detail. This can be excused under the pretence of recovering a plot from under a structural analysis: 'the structural account, which as a structural account must be common to many overlaying texts, simply did not give me enough to go on', I could claim. 'The structural analysis indicated the structure of an articulation, but did not reveal its actual elements'.

But this is grossly false. The problem is not underdetermination. If the primary, secondary, tertiary articulations, along with their transformations and refinements, if all these had been underdetermined, we could have simply filled in the gaps, and added arbitrary determinations consistent with those specified, until some mythical content corresponding to the mathematical structure would have been derived. The trouble is that the situation is exactly opposite. Gödel's articulations are so incredibly overdetermined, that one would be hard-pressed to find narrative features that could be successfully disguised as non-mathematical, and that would still emulate the given structure.

If my myth were a genuine plot, it could work its way around the problem. It would rely on an element mentioned in Forster's text: **mystery**. I would recount how the elements of the world are known only to the ancients or the gods, or perhaps how the arcane book coding these elements had been lost or destroyed, leaving the recounters of the myth with mere fragments of knowledge forlorn. But this is precisely a technique that would set the emerging plot apart from the mathematical text. The mathematical text insists on providing elements of structure, which it itself admits are not important. In section 6 of the 1934 text Gödel describes how only very few characteristics of a formal system are required to carry out his proof; and yet Gödel chooses to conduct the proof in an explicit formal system, and to painstakingly verify that it obeys all the stated characteristics. A touch of **mystery**, which a good plot couldn't do without, is exactly what

the mathematical text attempts to eradicate.

Such eradication attempts are of course unsuccessful. Mystery is prevalent in mathematical texts in many ways: from the awe-struck expression of the layman facing mathematical formulas; through the incredibly many details which are omitted to keep proofs readably-long; on to keeping some of the most important links of an argument to the 'end', where a proof will be cathartically completed; culminating with the statement of open problems, which the text leaves for the reader to confront.

But while in narrative texts mystery can function as an end in itself, for the mathematical text as it operates in contemporary academic mathematical circles an unsettleable mystery often functions as a hindrance or threat. This is the very threat that makes Gödel produce reasons to argue that, while the statement labelled $S(z_p, z_p)$ is unprovable and irrefutable, it must be marked as true. Other logicians resolve the same difficulty by stating that the predicates 'true' and 'false' simply do not apply to the statement marked $S(z_p, z_p)$. This again pushes mystery aside. On the other hand, 'It has a definite truth value, which we could never know' is a mystery-endorsing position, which contemporary mathematical texts appear to refuse.

3.3. And it spirals away from Gödel's proof

So far I located the gap between the mathematical text and mythical texts or plots in the former's fetish for justification (an operative **why?**), and in its aversion from explicitly endorsing mystery. The generality of these characterisations is correlated with their triviality. My demonstration may have its merits, but these conclusions alone do not justify as lengthy a story as I've concocted. My point is not these banal generalities. The devil, as is well known, is in the details. Let us survey some of them.

The first articulation in the myth presents a chart of elements. The first articulation in the proof provides a list of signs. But elementary substances and signs occupy different functional positions. They obey different grammars, at least on an 'ordinary' level. That Gödel's proof is making a (more or less successful) effort at working across this gap, namely at objectifying signs without destroying their signification, is not to be taken

lightly.[10] If my myth were to respect this endeavour, it should have been a semiotic myth concerning the vicissitudes of writing (in the generalised sense of making signs). Such myths exist — the Egyptian myth of Theuth and Thamus, which Plato invokes in the Phaedrus (and Derrida (1993) in *Plato's Pharmacy*) is an example. But to tell a story where Thamus instructs Theuth in the dangers of making signs by telling him a Gödelian story about story-telling (which is some form of making signs) would be no more than to provide a dramatic setting for Gödel's proof — a far cry from its pretended transubstantiation into a myth.

The text's second articulation arrays primitive signs to produce formulas, or, in mythical terminology, collects elements to form creatures. Three components of this articulation received special attention: negation/nemesis, substitution/feeding-impregnation and binding/endowing-with-phallus. I do consider the confounding of universal quantification and symbolic castration an interesting direction to pursue. But in order to allow this structural parallelism to emerge, the story had to be swept away from the structure of the proof. Free variables, which can be substituted or bound, were reconstructed as cavities, which could be fed, impregnated, or replaced with a phallus. How did two become three? The culprit is the articulation of the homogeneous position of free variable (any free variable has the same status as, and is replaceable by, any other)[11] into two distinct positions: vagina and mouth. The personification (or Golemisation) of formulas demanded it.

Of course, this structural deformation could have been avoided. The cavities could have been homogeneously marked as ears or nostrils. But to do so would be a very early confession of failure. Which cave-dwelling audience would sit through a story about creatures plugging their ears with names or shoving lineages up their noses? Which reader of plot would not call the arbitrary nostril combinatorics for what it is: a thinly veiled math-

[10] I believe that this effort is a feature of a large portion of modern mathematics, rather than a unique feature of Gödel's text — the very feature that allows mathematics to support both realist and formalist philosophies. This effort is also a feature of structuralism, culminating in an implosion from which post-structuralism erupted. The first point will be treated in the third chapter of this book. The second point will only be skimmed over in the next subsection.

[11] All free variables relevant to the story are of the same logical *type*.

ematical text. The semantically constrained combinations and the content restrictions that a myth or narrative must endure limit the capacity to transform proof into myth.

Then appears the transformation internal to the second articulation: the homology between creatures and names. There I suppressed the fact that in Gödel's proof every name (arithmetic number), which is represented formally (a numeral in the formal system), has, again, its own arithmetic representation, which can, again, be formally transformed into a different numeral representing it in the formal system. My myth may have a fair degree of incestuous and cannibalistic features, but it doesn't even begin to measure up to the criss-crossing and self-reflective features included in Gödel's tour-de-force.

The third articulation, introducing proof/birth, includes another important incongruity. Mythical creatures tend to have a unique lineage (or, at most, a few alternative accounts of lineage). But a formula, if it can be proved at all, must have infinitely many different proofs. This, again, raises the object-sign tension, which my myth tries to suppress. When a formula is introduced with different proofs, is it still the same formula? When I substitute a numeral indicating a formula into a free variable contained in that same formula, are the 'substituted' and 'host' formula the same formula?

The cost of restricting my mythical entities to occupy object-like, rather than semiotic functional positions was the transcription of semiotic problems onto a different locus. I had to insist that a fed or impregnated Golem is altogether different (differently named and differently begotten) from an unfed or unimpregnated one, rather than consider them as the same creature, which has undergone some non-essential modification.

In fact, the entire structure of signification, internal to Gödel's argument, is obliterated from within the myth at a high cost, and imposed upon it as a secondary, external function, through the question 'what does the myth signify?'. This disappearing-reappearing act is, to some extent, a general feature of structural analyses: structures tend to flaunt a certain idiosyncratic, self-regulating reflexivity (they are what they mean), which places them in a paradoxical position relative to that which they are supposed to structure, and makes it difficult for them to live up to their *raison-d'être* as structures-of.

By the time the refined version of the third articulation was to be included, my capacity as story-teller was already completely exhausted (but I doubt that other story tellers could 'stay in the ring' more than a couple of rounds longer). Within the articulated construction of formulas, a privileged class of formulas is demarcated: formulas that can be expressed in terms of recursive functions, and whose provability can therefore be decided by a finite algorithmic computation. This privileged class was mythically reconstructed as oracles: magical creatures, whose birth can be divined, and further indicates the begetting-possibility of other creatures to other lineages. I couldn't find any mythical excuse for their special role, and if one were to read carefully, I couldn't even justify their necessity. After all **We**, who have full knowledge of the third articulation, can already determine whether a given creature can be born to a given lineage (without assuming, of course, to know whether a given creature could be born to any lineage, as this would require checking whether it can be born to each and every possible lineage — an infinite task). Why would **We** need an oracle? One could come up with an excuse (say, **We** wanted to allow lower entities to access this knowledge, or to simplify the process of acquisition of this knowledge). But to do that would be, again, either to force more implausible features on the myth, or to let the myth follow its own thread, losing its footing in the mathematical text.

Eventually, the finalé comes. The creation of Qew and its variations, and the discovery of disharmony that these creatures entail. At this point, I believe, even the most cooperative reader would be let down. After all my contorted efforts one is left with nothing but a thinly veiled logical riddle. The details of mismatch are fundamental: temporality is differently structured in the myth and in the proof, the plot involves no births except those of oracles (whose role was supposed to be restricted to the prediction of 'normal' births, completely absent from the myth), and I eventually chose to confine the syntactic argument to the margin in order not to drag the reader through an even more monstrous cacophony.

3.4. And away from structure

The myth is forced to follow the limitations imposed by the restrictive devices of myth and plot away from Gödel's proof. Reflexively complex functional positions in the proof are violently suppressed. The transformation of proof to myth fails to restrict its impact to the elements, and violates structure as well. To quote Jean Starobinski (paraphrasing de Saussure's account of mythical transformation): **In the structural order of narrative, the basic substance of symbolism is not only used but also modified. For structure itself can be modified and become a modifier. If one simply varies the "external" links of the original material, apparently "intrinsic" characters become entirely changed** (Starobinski 1979, 6). And my experiment is but one more demonstration of a how a transformation, which is supposed to be structure preserving, carries structure away.

But even after all these sacrifices, our story still doesn't sound like a proper myth, or even a proper story for that matter. The restrictions of the mathematical proof, the logic of its persistent **why?**, even though they were not properly respected, were still violent enough to constrain the myth to the extent that it can no longer be identified as myth. Mathematical traces arrange **events with emphasis on causality**; **the ground plan is a plot, and the characters are ordered to acquiesce in its requirements** ... **characters are involved in various snares, they are finally bound hand and foot** ... **and yet, for all the sacrifices made to it, we never see action as a living thing** ... **the characters have been required to contribute too much to the plot** ... **their vitality has been impoverished, they have grown dry and thin**. We have **emphasised causality more than** the **medium permits** (Forster 1927, 89–90).

Our product, a narrative stifled under an imposing logical structure, Forster explains, is typical of the novels of his time. **Sometimes a plot triumphs too completely. The characters have to suspend their natures at every turn, or else are so swept away by the course of Fate that our sense of their reality is weakened** ... **In the losing battle that the plot fights with the characters, it often takes a cowardly revenge. Nearly all novels are feeble at the end. This is**

because the plot requires to bound up ... **and usually the characters go dead while** the novelist **is at work, and our final impression of them is through deadness** ... **most novels do fail here — there is this disastrous standstill while logic takes over the command from flesh and blood** ... **nothing is heard but hammering and screwing** ... **The characters have been getting out of hand, laying foundations and declining to build on them afterwards, and now the novelist has to labour personally, in order that the job may be done on time. He pretends that the characters are acting for him. He keeps mentioning their names and using inverted commas. But the characters are gone or dead. The plot, then, is the novel in its logical intellectual aspect: it requires mystery, but the mysteries are solved later on: the reader may be moving about in worlds unrealised, but the novelist has no misgivings** (Forster 1927, 89–92).

The logical aspect of plot, as Forster articulates and our experiment verifies, is exactly that which a story cannot take upon itself without losing its vitality. No wonder, then, that my little exercise did not succeed in turning proof into living narrative. I believe this here is an indication of inherent incapacities that structural analyses impose. But this indication alone does not rule out, of course, that one *could* generate non-narrative texts, which *would* be structurally analysable into Gödel's proof, and *would*, perhaps, lead, in some obscure way, to claiming that mathematics is indeed the structure of philosophy, thought or the universe. I will not deprive such hopes from those who depend upon them. But since I am going to read Gödel's proof with tools designed mostly in the context of analysing literary texts, it is important, as a means of precaution, to expose and explicate the tensions between these two kinds of texts.

That every mathematical argument is accompanied by narrative is difficult to doubt. But that such narrative underlies the argument, or is structurally codified inside it, is far too strong a claim. Should one study which similarities can and cannot be produced between different texts, and which transformations can force one text to become another? Undoubtedly, this is worth studying, as long as one bears in mind that the similarities are imposed, not exposed, and that the transformations do not emerge, but are rather projected. And when extracting narrative from a mathematical text,

one should remember that (as in the case of de Saussure's quest for hidden messages in Latin poetry) **Every complex structure furnishes an observer with a range of elements which will allow him to choose a *sub-ensemble* apparently endowed with sense, and which nothing prohibits *a priori* a logical or chronological antecedence** (Starobinski 1979, 44). The derivation of structure is *constrained* by the structured phenomena, but doesn't quite *retain* an 'original' structure. Could this functional oddity be related to the possibility that structure does not simply *exist*, but attains some less straightforward ontological position?

That both mathematical and narrative texts are phenomena of language is hard to deny. But to study the process of semiosis in one through a corresponding process in the other would jump the gun. The basic processes of semiosis may or may not be universal, but their specificities deserve to be concretely studied within each discourse where they make their mark. In this book I therefore analyse semiosis in Gödel's mathematical text with tools that have originally been designed for the study of literary texts, but in doing so I do not assume that mathematics is literature. **In the novel**, Forster writes, **all human happiness and misery does not take the form of actions, it seeks means of expression other than through the plot, it must not be rigidly canalised** (Forster 1927, 90–91). This essay merely aims to show that in the mathematical text, all meaning does not take the form of logic, it seeks means of expression other than through inference rules, it must not be rigidly canalised.

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